Geometric nonlinear analysis of laminated composite flat stiffened panels using Variational Asymptotic Method (VAM)

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Nomenclature

δ	Small parameter identification
$\epsilon_{11}, \epsilon_{22}$	Extensional 2-D strains
ϵ_{12}	In-plane 2-D shear strain
γ_{13}, γ_{23}	Transverse 2-D shear strains
Γ_{ij}	Components of green strain
κ_{11}, κ_{22}	Bending 2-D curvatures
κ_{12}	Torsional 2-D curvature
$\lambda_i, \lambda_4, \lambda_5$	Lagrange multipliers
ω_i, v_i	Warping 3-D components in zeroth and first order approximation
ϕ_1, ϕ_2	Rotations along x and y coordinates
Π_0, Π_1	Zeroth and first order energy
ψ	Strain energy density function for an individual layer
σ_{i3}	Second Piola-Kirchhoff stress components
φ_i	Body force components
ξ, η	Natural coordinates
A, B, D	Sub matrices of 2-D stiffness matrix
C_{ij}	Components of direction cosine matrix
$Cb_{ij}, Cb_{ij}, Ct_{ij}$	Material constants of bottom, core and top layer
e_{ijk}	Permutation tensor components
F_{ij}	Components of deformation gradient tensor
F_r, M_r, T_r	Force, moment and transverse shear stress resultants
g	Determinant of metric tensor in undeformed configuration
h_b, h_c, h_t	Thickness of bottom, core and top layers
I_{ij}	Identity matrix components
N_k	Shape functions

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P, P^*	Position of any arbitrary material point in undeformed and deformed configurations
T_i^t, T_i^b	Top and bottom surfaces traction forces
U	Total strain energy per unit mid surface area
u, v, w	Translations along x, y, z coordinates
W	Principle of virtual work
y_i	Cartesian coordinates
$\widehat{\mathbf{r}}, \widehat{\mathbf{R}}$	Position vectors in undeformed and deformed configuration
$\mathbf{b}_i, \mathbf{B}_i$	Base vectors in undeformed and deformed configurations
$\mathbf{G}_i$	Covariant base vectors in deformed configuration
$\mathbf{g}_i, \mathbf{g}^i$	Covariant, contra-variant base vectors in undeformed configuration
$\mathbf{r}, \mathbf{R}$	Position vector points on the reference surface in undeformed and deformed configuration

I. Introduction

Stiffened composite plates are efficient and reliable structures. By adding small weight in terms of stiffeners to the plates, total strength of the structure can be increased predominantly [1]. However, the structure fails [2] majorly in the presence of compressive loads, transverse loads [3]. The stiffened structure analysis is one of the challenging problem in engineering domain due to the combination of plate and stiffener models [4], various material properties and different boundary conditions. The stiffened structures are analyzed in various approaches; Sheinman[5] presented the nonlinear analysis of laminated stiffened composite panels and developed nonlinear equations which are solved by using finite difference method under various boundary conditions. Patel et al.[6] analyzed the stiffened panel nonlinear behavior under various boundary conditions. Sheikh[7] investigated the geometric nonlinear analysis of stiffened plates by using spline finite strip method. Several experimental tests have been conducted by Romeo[8] and Park [9] on blade and hat stiffened panels made up of graphite/epoxy material subjected to uniaxial compression and compared the accuracy with theoretical analysis.

Here, the state-of-art Variational Asymptotic Method (VAM), which was introduced by Berdichevsky [10], is applied to develop a computer code NASSVAM (Nonlinear Analysis of Stiffened Structures using Variational Asymptotic Method). Further, VAM is used to analyze the behavior of stiffened structures under compressive and transverse loads. This method is computationally efficient and gives us asymptotically correct solutions. Berdichevsky [10] is the first researcher to apply VAM technique to model shells. VAM method is developed by identifying the geometric and physical small parameters that are inherent to the problem definition. Atilgan et al. [11]; Sutyrin and Hodges [12]; Harusampath [13] exploited this VAM approach to the various applications. Le and Nguyen [14, 15] developed the analytical formulations for beams. Further, Le and Yi [16] and Le [17, 18] have established the error estimate of the approximate laminated and functionally graded plates and shells showing the accuracy of variational asymptotic method. The small parameters are utilized to classify the total potential energy in an asymptotic manner [19]. These energy density functionals are minimized using calculus of variations.

In the present work, the dimensional reduction of 3-D laminated plate is carried out using VAM [11], and this procedure is applied to the 3-D energy functional to reduce its dimension to an equivalent 2-D plate [20]. First variation of energy functional with respect to the unknown warping functions will provide a set of differential equations that are solved by adopting the appropriate boundary conditions [21]. As a result, an asymptotically accurate analytical expressions for warping functions are obtained [22]. Then, substitute back these warping functions into the strain energy and integrate through-the-thickness. The double derivative of 2-D strain energy density with respect to 2-D generalized strains will give the 2-D nonlinear constitutive law for laminated composite flat plate.

The 2-D nonlinear program [23] is developed by taking VAM input (2-D constitutive law). The integrated model of stiffened structure nonlinear behavior is analyzed by obtaining load-displacement curve with the numerical calculations and the results shows good agreement with those available in the literature.

II. 3-D kinematics

Consider a 3-D plate composed with a set of laminae, called as laminated plate, with relatively small thickness, h as compared to its length and width dimensions. Therefore, plate is represented as 2-D reference surface and here mid-surface is considered as its reference surface. The un-deformed and deformed reference surface of a plate can be represented in the cartesian coordinate system y_α, where y_α denotes the surface coordinates ($\alpha = 1, 2$) and $y_3 = h\xi$ uniquely represents the normal coordinate at any arbitrary point in the 3-D continuum medium, where $-1/2 < \xi < 1/2$. Through out the formulation, latin indices assumes 1, 2 and 3 while greek indices are 1 and 2, further, repeated indices are summed over their ranges. Let b_i denote the orthogonal unit vectors in the un-deformed plate configuration along y_i, one can express the position of any arbitrary material point $P(y_1, y_2, y_3)$ by its position vector $\hat{\mathbf{r}}$ in the un-deformed plate configuration.

$$\widehat{\mathbf{r}}(y_1, y_2, y_3) = \mathbf{r}(y_1, y_2) + y_3\mathbf{b_3} \tag{1}$$

Covariant and contra-variant base vectors are expressed as: $\mathbf{g}_i = \partial\hat{\mathbf{r}}/\partial y_i$, $\mathbf{g}^i = \frac{1}{2\sqrt{g}} e_{ijk}\mathbf{g}_j \times \mathbf{g}_k$, where e_{ijk} represents the permutation tensor and $g = \det(g_i.g_j)$. The position vector in the undeformed configuration can be illustrated from a fixed point O to the material point P, as shown in Fig.1. The deformed plate configuration is described by representing the deformed position vector [24] $\widehat{R}$, which was at $\widehat{r}$.

$$\widehat{\mathbf{R}}(y_1, y_2, y_3) = \mathbf{R}(y_1, y_2) + y_3\mathbf{B_3} + \omega_i(y_1, y_2, y_3)\mathbf{B}_i(y_1, y_2) \tag{2}$$

where $\mathbf{R}$ is expressed as: $\mathbf{R}(y_1, y_2) = \mathbf{r}(y_1, y_2) + u(y_1, y_2)$. The material point in the deformed configuration is represented as P^*, from the same fixed point O, in the base coordinate system and u is the displacement field. $\mathbf{B}_i$ is represented as the deformed configuration unit vectors. $\omega_i(y_1, y_2, y_3)$ are 3-D warping field components, where ω_1 and ω_2 are in-plane warping functions and ω_3 is out-of-plane warping function. Thus, the variation of normal through the thickness is accounted in the current VAM formulation by introducing the warping function and these can be solved by using appropriate constraints. The relation between base vectors $\mathbf{B}_i$ and $\mathbf{b}_i$, are represented in the deformed and undeformed configurations respectively, can be specified as: $\mathbf{B}_i = C_{ij}.\mathbf{b}_i$, where $C_{ij} = \mathbf{B}_i.\mathbf{b}_i$. In the deformed configuration, covariant base vectors are specified as: $\mathbf{G}_i = \partial\hat{\mathbf{R}}/\partial y_i$. Thus, the above expressions are utilized to form the appropriate strain that is Green stain and its components are represented as: $\Gamma_{ij} = (F_{ik}F_{kj} - I_{ij})/2$, where I_{ij} is identity matrix components with size of 3×3 and F_{ij} is the Deformation Gradient Tensor (DGT), is expressed as: $F_{ij} = \mathbf{B}_i.\mathbf{G}_k\mathbf{g}^k.\mathbf{b}_j$.

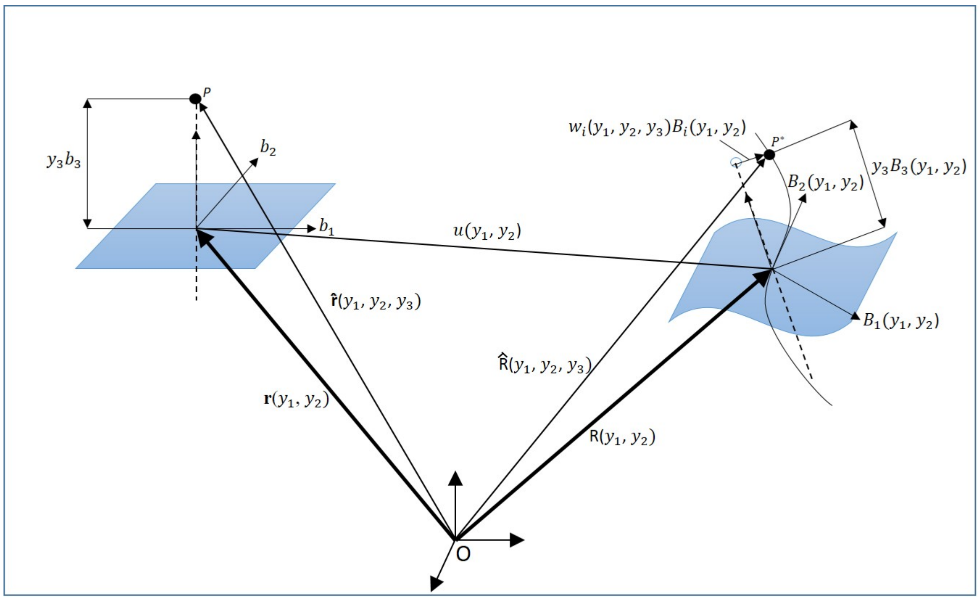

Figure 1. Schematic representation plate deformation

III. Potential energy

The principle of minimum potential energy is applied and set the first variation of this energy to zero in order to determine the unknown warping functions. The corresponding structural deformations must satisfy the minimum potential energy principle. Thus, obtain the stationary points of the potential energy functional as displacements with respect to the imposed global and inter-laminar constraints. As a result, the boundary value problem can be formed but it is so complex to solve unknown functions due to the presence of coupled nonlinear differential equations. Therefore, the analytical expressions for 3-D warping functions can not be achieved directly. However, through VAM procedure one can obtain the solutions of these warping functionals in an asymptotically accurate manner, by taking the advantage of small parameters that are inherent to the problem definition. Total strain energy (U) per unit mid surface area is obtained by, the integration of 3-D strain energy density function (ψ) for all laminae over their thicknesses and summation of all the layers contribution.

$$U = \int_{-\frac{h_c}{2}-h_b}^{-\frac{h_c}{2}} \psi \, dy_3 + \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \psi \, dy_3 + \int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_b} \psi \, dy_3 \tag{3}$$

where $\psi = (\sigma^T \Gamma)/2$, σ and Γ are the components of 3-D stress and strain fields respectively. For a 3-D geometrically nonlinear problem, total potential energy function can be specified in terms of total strain energy U and total work done W (principle of virtual work), is given as: $\Pi = U - W$. The total work done by the applied forces acting on the bottom and top surfaces of the structure, and body forces are specified as: $W = T_i^t \omega_i^t + T_i^b \omega_i^b + < \varphi_i \omega_i >$, where T_i^t, T_i^b are the top and bottom surfaces traction force, φ_i is the body force, ω_i^t and ω_i^b are warping field components of top and bottom surfaces. Throughout the paper angular brackets denotes through-the-thickness integration of Laminated Composite Stiffened Panel (LCFSP) structure at any given point of location on the mid-plane.

IV. Constraints on warping functions

The warping field of each lamina can have different forms of functional to make the equation of $\widehat{\mathbf{R}}$ to be determinate, the following constraints have been imposed on the warping functions to avoid the redundancy.

$$\langle \omega_i(y_j) \rangle = 0 \text{ and } \langle y_3 \omega_\alpha(y_j) \rangle = 0 \text{ where } i, j = 1, 2, 3 \text{ and } \alpha = 1, 2 \tag{4}$$

The above constraints are written for a laminate with three layers as:

$$\int_{-\frac{h_c}{2}-h_b}^{-\frac{h_c}{2}} \omega_i^b(y_j) \, dy_3 + \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \omega_i^c(y_j) \, dy_3 + \int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_b} \omega_i^t(y_j) \, dy_3 = 0 \tag{5}$$

$$\int_{-\frac{h_c}{2}-h_b}^{-\frac{h_c}{2}} y_3 \, \omega_\alpha^b(y_j) \, dy_3 + \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} y_3 \, \omega_\alpha^c(y_j) \, dy_3 + \int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_b} y_3 \, \omega_\alpha^t(y_j) \, dy_3 = 0 \tag{6}$$

where h_b, h_c and h_t are the thickness of bottom, core and top layers. Either it is multi-functional composite laminate[25] or a composite laminate, it must satisfy the continuity conditions at the interfaces of all layers within the laminate. The continuity conditions are explained physically by applying the continuity of warping filed and transverse stresses at the interface of layers. Mathematical expressions of the continuity conditions are expressed as:

$$\omega_i^t = \omega_i^c \mid_{y_3=\frac{h_c}{2}} \text{ and } \omega_i^b = \omega_i^c \mid_{y_3=\frac{-h_c}{2}};\ \sigma_{\alpha 3}^t = \sigma_{\alpha 3}^c \mid_{y_3=\frac{h_c}{2}} \text{ and } \sigma_{\alpha 3}^b = \sigma_{\alpha 3}^c \mid_{y_3=\frac{-h_c}{2}} \tag{7}$$

V. Dimensional reduction of LCFSP structure using VAM

The process of reproducing the 3-D strain energy stored in a three-dimensional structural body to an equivalent reference surface or a mid-surface formulation (2-D) is referred as a methodology of dimensional reduction process [26]. In this process, the major work is to reproduce the total strain energy distribution of a 3-D body over an equivalent 2-D body and this procedure cannot be accomplished exactly. Nevertheless, the reduced formulations can be developed according to the VAM procedure in an asymptotically accurate manner. Plates and shells are considered as dimensionally reducible structures because of its very small thickness as compared to the other two planar dimensions[4]. Here, VAM applied by taking the advantage of small parameters (thickness to maximum wavelength ratio and strains) that exists within the problem definition and not making any adhoc kinematic assumptions. The present model is also analyzed as a laminated plate using VAM and followed the procedure, is given below.

3-D laminated composite flat stiffened panel can be represented as 2-D model, which is achieved in an asymptotically correct and computationally efficient approach. Further, the order of magnitude of small parameters can be determined and these can influence the entire formulation. The estimated order of magnitudes for small parameters pertaining to the LCFSP structure has been segregated according to the leading order terms and the functional to be minimized based on those estimated orders. Therefore, the total potential energy can be written with different order sets such as: $\Pi = \Pi_0 + \Pi_1 + \Pi_2 ...$ where Π_0 is the zeroth order potential energy, Π_1 is the first order potential energy and so on. The sequence of potential energy based on the order of magnitude is as follows: $\Pi_0 \gg \Pi_1 \gg \Pi_2 ...$ and higher order terms are less critical for the engineering application.

The 3-D strain energy expression comprise of all terms (2-D strains $\sim O(\delta^2)$, warping functions $\sim O(\delta^6)$ and its derivatives $\sim O(\delta^2)$) up to $O(\delta^4)$ in the zeroth order approximation, Π_0, where δ is used to assess the order of magnitudes. This Π_0 corresponds to the major energy contribution and is expressed in an asymptotically accurate approach. The next higher order terms $O(\delta^6)$ are considered for the first order approximation. The unknown 3-D warping functions are determined by the minimization process of total potential energy functional, is given as:

$$\delta\Pi^* = 0, \text{ where } \Pi^* = \Pi - \Lambda_i \langle \omega_i \rangle - \Lambda_4 \langle y_3 \omega_1 \rangle - \Lambda_5 \langle y_3 \omega_2 \rangle \tag{8}$$

and Λ_i $(i = 1, 2, 3)$, Λ_4 and Λ_5 are the Lagrange multipliers. The analytical expressions of warping functions and 2-D stiffness coefficients of zeroth and first order approximations are given in the Appendix.

VI. 2-D plate nonlinear analysis

Stiffened panel 2-D analysis starts with the input obtained from the 1-D through-the-thickness analysis, those inputs are in terms of material constants $(Cb_{ij}, Cb_{ij}, Ct_{ij})$, 2-D strains $(\epsilon_{11}, \epsilon_{22}, \gamma_{12}, \kappa_{11}, \kappa_{22}, \omega, \gamma_{13}, \gamma_{23})$ and thickness coordinate (y_3). In addition, the input parameters are nodal coordinates, material properties, loads and boundary conditions. The defined shape functions for 4-node iso-parametric quadrilateral elements and its derivatives are substituted in the 2-D kinematics formulation.

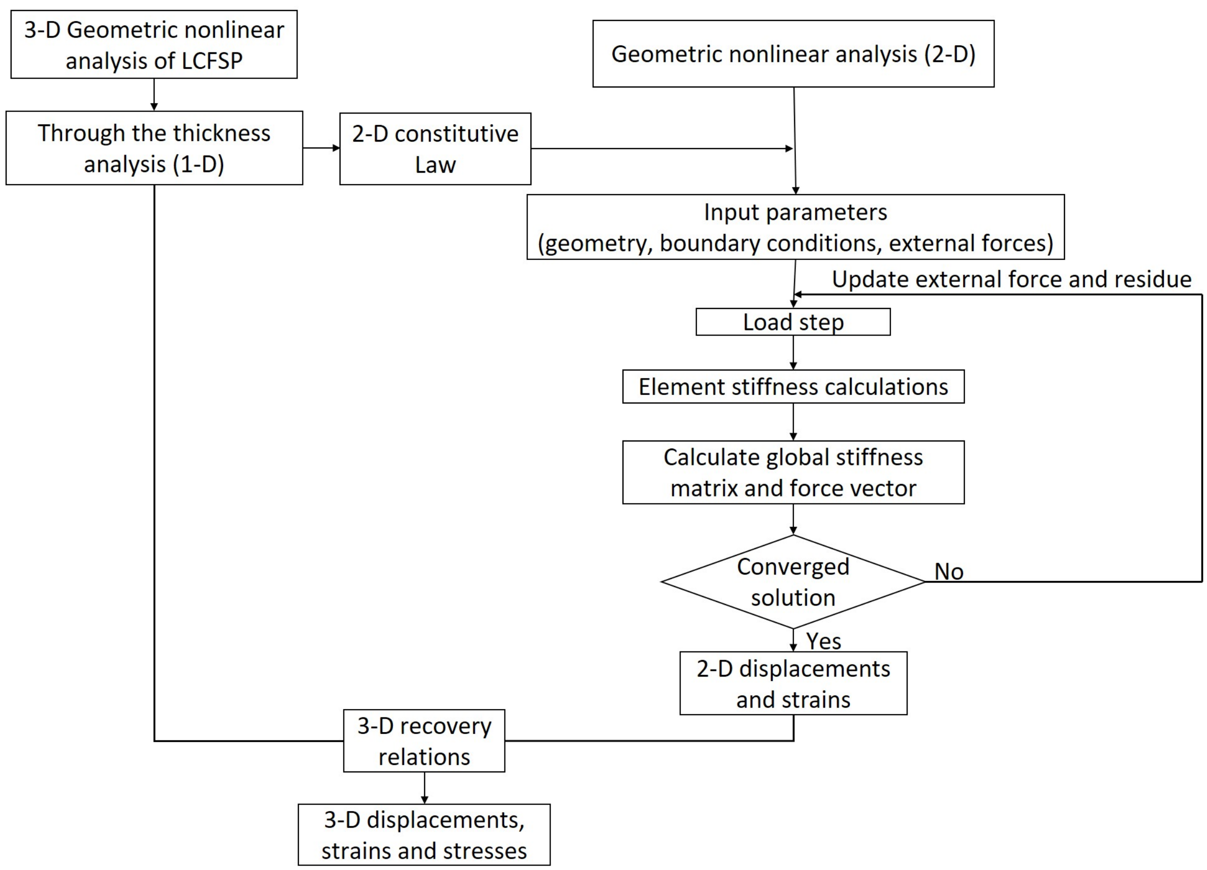

Figure 2. Flow chart of geometric nonlinear analysis (NASSVAM)

The 2-D geometric nonlinear strain energy density functional is formed after substitution of these terms in the kinematic expressions. Thereafter, the internal forces are obtained by taking first derivative of strain energy density functional with respect to the nodal displacement variables and double derivative of strain energy density function with respect to the nodal displacement variables to obtain elemental stiffness matrix. Subsequently, external forces can be computed to solve the required nonlinear equations (force-displacement equation) by using Newton-Raphson iterative method in order to obtain 2-D nodal displacements, as shown in Fig. 2. Four-noded isoparametric plate elements have been used for discretization. The deformation fields are interpolated using shape functions and those are expressed as: $u = \sum_{k=1}^{4} N_k u_k$, where u_k are the nodal displacements, N_k are the linear shape functions. The functional form of the shape functions are expressed as: $N_k = (1 \pm r)(1 \mp s)/4$, where r and s are the natural coordinates. Further, the 3-D displacements can be calculated with the inputs of the 2-D results through closed-form 3-D recovery relations, which are derived as part of the through-the-thickness analysis. The recovery relations for 3D displacements field are expressed as:

$$U_i = u_i + y_3\theta_i + C_{ij}(w_j + v_j) \quad \text{where} \ \ (i, j = 1, 2, 3) \tag{9}$$

where u_i are the 2-D plate displacement variables, θ_i are the rotations of the normal (2-D finite element analysis provides reference surface nodal displacements and rotations in three global coordinate directions) and y_3 is the thickness coordinate. The final 3-D displacements are due to the contribution of 2-D displacements, rotations and, local rotation and stretching/compression of the normal (through the asymptotically accurate warping components of zeroth-order and first-order perturbations $(w_j + v_j)$ with the appropriately transformed by the direction cosine matrix C_{ij}).

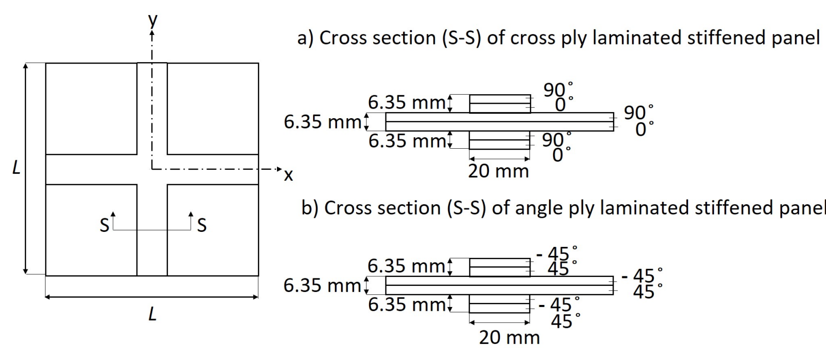

Figure 3. Schematic representation of stiffened laminated plate cross section a) Cross ply b) Angle ply

A. Ply-drop

Laminated plates with ply-drop are generally used in the applications where the compressive loads are prominent[27]. In the present work, it has been implemented in three stages, first stage; obtain the 2-D nonlinear constitutive law for each laminate individually through the thickness analysis that can be utilized as input for the LCFSP 2-D nonlinear finite element analysis. In detail, analytical expressions of 2-D constitutive law (stiffness matrix) can be obtained for three layer and two layer laminate of skin respectively as well as for the corresponding two layer laminate of stiffener. The substitution of input values in the analytical expression gives a numerical stiffness matrix, where the input values are in terms of 2-D strains, material constants and thickness of the laminae.

In second stage, identify the elements that are associated with the specific laminate to the specific geometry including the skin and stiffener. Thereafter, the element loop can facilitate to take the required constitutive law for the described elements. In third stage, assign an appropriate 2-D constitutive law to elements in order to achieve the ply drop accordingly for LCFSP. The coordinates of a stiffener are transformed to skin coordinate system to elucidate the integrated structure. Then, the developed transformation matrix is applied by pre and post multiplication with elemental stiffness matrix.

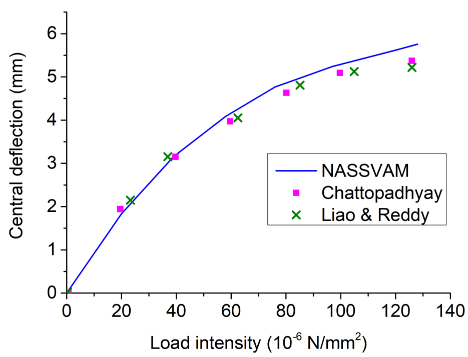

Figure 4. Central deflection of cross ply laminate

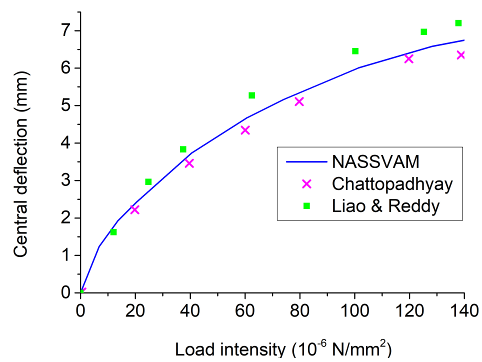

Figure 5. Central deflection of angle ply laminate

VII. Results and validation

A. Angle ply and cross ply laminated stiffened structures

A rectangular composite stiffened plate is analyzed by using the present NASSVAM (Nonlinear Analysis of Stiffened Structures using Variational Asymptotic Method) approach. The geometry of a simply supported laminated composite stiffened panel is shown in Fig. 3. The current model has been analyzed for angle ply and cross-ply laminate. The stacking sequence for cross ply laminate is showed in Fig. 3a and the angle ply laminate stacking is depicted in Fig. 3b. The panel length (L) is 2438 mm, the stiffener height and width are S_h = 6.35 mm and S_w = 20 mm respectively.

This problem has been analyzed by taking one quarter of stiffened plate due to its symmetry. To develop the FE model of stiffened panel, the information such as, material properties [28], dimensions, boundary conditions, mesh description, analysis method and type of assumptions made for the analysis can influence the results [29]. In the present problem, plate and stiffener material properties are, $E_{11} = 25\ E_{22}$, $E_{22} =$ 7031 MPa, $G_{12} = G_{13} = 0.5\ E_{22}$, $G_{23} = 0.2\ E_{22}$ and $\nu_{12} = 0.25$. The transverse load is applied on top side of the plate and simply supported boundary conditions along the four edges of the stiffened panel. For cross ply laminate problem the boundary conditions are given below as BC1:

$u = w = \phi_2 = 0$ at y=a/2, $u = w = \phi_1 = 0$ at y=a/2, $u = \phi_2 = 0$ at y=0, $v = \phi_1 = 0$ at x=0

and corresponding to the angle-ply laminate problem boundary conditions are BC2:

$u = w = \phi_2 = 0$ at y=a/2, $u = w = \phi_1 = 0$ at y=a/2, $u = \phi_2 = 0$ at y=0, $u = \phi_1 = 0$ at x=0

where u, v, w are translations along x, y, z coordinates and ϕ_1, ϕ_2, ϕ_3 are the rotations along x, y and z coordinates. Each node associated to the finite element has six degrees of freedom. The choice of convergence criteria and the associated convergence tolerance has been chosen carefully in Newton-Raphson iterative procedure. The results are validated with the reported data by Liao and Reddy [30] and Chattopadhyay [31]. The load intensity verses central deformation curve for angle ply lamination is depicted in Fig. 5. The present results have good agreement with the published results and close to the results of Chattopadhyay [31]. The central deflection curve for cross ply problem is shown in the Fig. 4 and the results are matching well with the results of both Chattopadhyay and Liao and Reddy. However, it is observed that the central deflection of cross ply problem is less when compared to angle ply problem because, the cross ply laminate is stiffer in bending as compared to the angle ply laminate [32].

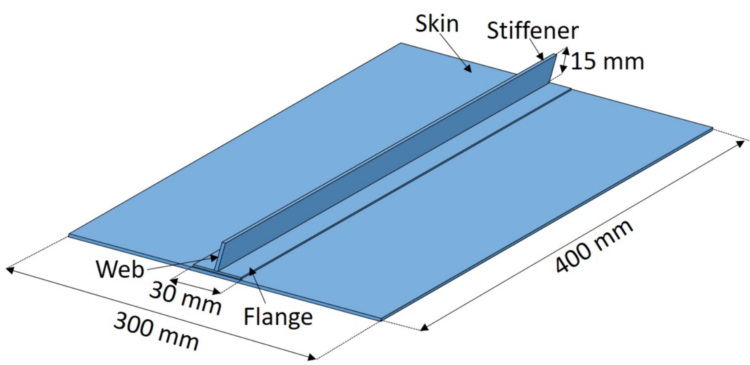

Figure 6. Representation of LCFSP structure with skin, flange and blade stiffener

B. LCFSP structure with ply-drop

The stiffener and skin are analyzed by considering both as plate models with the 2-D displacement continuity at their interface. In the laminated composite stiffened structural design, the skin constitute of two laminae with each laminae thickness of 0.736 mm, the flange is made up of three laminae with each lamina thickness of 0.736 mm, and stiffener laminate comprise of two layers with each lamina thickness of 0.736 mm. The ply-drop is applied between flange and skin by assigning the required constitutive law to the corresponding elements. The geometry of LCFSP model is represented with a length of 400 mm, width of 300 mm and the web height as 15mm with flange width of 30 mm, Fig. 6. The stacking sequence for skin is [0/90], flange is [0/90/0] and web is [0/90]. The stiffened plate material properties are, E_{11} = 152800 MPa, $E_{22} = E_{22} =$

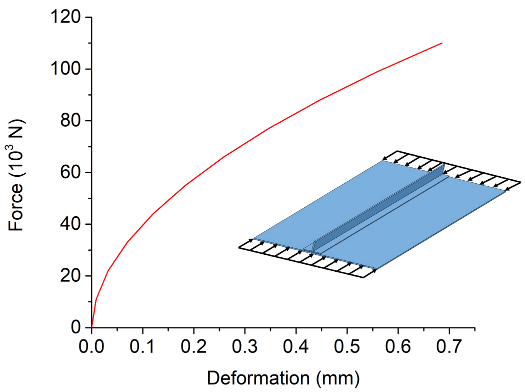

Figure 7. Force-deformation curve of LCFSP structure under compressive load

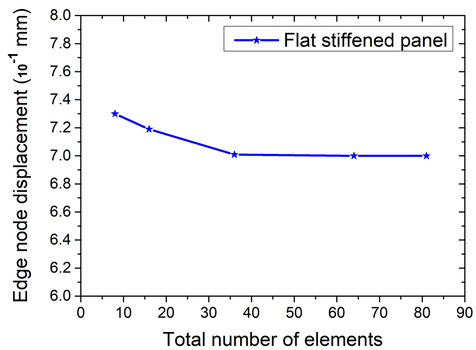

Figure 8. Mesh convergence study of flat stiffened panel

8700 MPa, $G_{12} = G_{13} = 4200$ MPa, $G_{23} = 3150$ MPa, $\nu_{12} = \nu_{13} = 0.335$, $\nu_{23} = 0.380$ and ply thickness $= 0.736$ mm. The mesh convergence study is carried out, as shown in Fig. 8 and sixty-four elements are taken (mesh density) to perform the geometrical nonlinear analysis. The boundary conditions are simply supported at the four edges of stiffened panel and the applied force is compressive force on two opposite edges along the width. The results are assessed as load-displacement response, as shown in Fig. 7.

VIII. Conclusion

A composite stiffened panel is modeled and analyzed using Variational Asymptotic Method (VAM). A symbolic mathematical computational tool, Mathematica® has been used to develop the theoretical formulation of VAM and developed a computer code NASSVAM for geometric nonlinear analysis of stiffened panel. In the development of VAM, 3-D stiffened panel problem is broken down into two stages; first stage (1-D analysis), perturbed warping functions are substituted in the strain energy expression in order to obtain 2-D constitutive law; second stage (2-D analysis), the obtained 2-D constitutive law from through-the-thickness analysis is provided as an input to the 2-D plate analysis (reference surface analysis). NASSVAM program has been developed using Newton-Raphson iterative method and used to analyze the behavior of flat stiffened composite panel. NASSVAM results are compared with the test cases available in literature for cross-ply stiffened panel and angle-ply stiffened panel. The results are showed good agreement with the literature results. Thus, the geometric nonlinear analysis of laminated composite stiffened panel is carried out by analyzing both the skin and the stiffener as plate models with the 2-D displacement continuity at their interface.

Appendix

The zeroth order warping functions are determined from the minimization process of total potential energy functional [Eq. 8]. The zeroth order warping solutions (for top surface as ω_1^t, ω_2^t and ω_3^t; for core surface as ω_1^c, ω_2^c and ω_3^c; for bottom surface as ω_1^b, ω_2^b and ω_3^b) are substituted back in the strain energy functional expression and integrated through the thickness to obtain the 2-D energy; the second partial derivatives of this strain energy with respect to 2-D strains, $\varepsilon^T = \{\{\epsilon\}, \{\kappa\}, \{\gamma\}\}$ (where $\{\epsilon\}^T = \{\epsilon_{11}, \epsilon_{22}, \gamma_{12}\}, \{\kappa\}^T = \{\kappa_{11}, \kappa_{22}, \omega\}, \{\gamma\}^T = \{\gamma_{13}, \gamma_{23}\}$), provides the generalized 2-D constitutive law. $F_r^T = \{N_{11}, N_{22}, N_{12}\}$, are the force stress resultants; $M_r^T = \{M_{11}, M_{22}, M_{12}\}$, are the moment stress resultants; $T_r^T = \{T_{13}, T_{23}\}$, are the transverse shear stress resultants; these are specified as:

$$\begin{Bmatrix} F_r \\ M_r \\ T_r \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & S \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \\ \gamma \end{Bmatrix} \tag{10}$$

where the sub matrices A, B, D are of size 3×3 and S is of size 2×2, the elements in these matrices represents the extensional, coupling, bending and shear stiffness coefficients respectively. These sub matrices includes: Cb_{ij} , Cc_{ij}, Ct_{ij} (material constants of bottom, core and top layers) and $h_b = h_c = h_t = h/3$. The 2-D strain components $\epsilon_{11}, \epsilon_{22}$ are in-plane (along y_1, y_2 directions) extensional strains and γ_{12} is the in-plane shear strain. κ_{11}, κ_{22} are the mid surface bending curvatures and ω is the twisting curvature. γ_{13}, γ_{23} are the transverse shear strain components and they act in a plane $y_1 - y_3$ and $y_2 - y_3$, finally the 2-D constitutive law is constituted with all these terms. The analytical expressions of zeroth order warping functions are expressed below:

$$\begin{aligned} \omega_1^t = &-\frac{1}{4h^2(\mathrm{Cb}_{55}(17\mathrm{Cc}_{55} + 47\mathrm{Ct}_{55}) + 17\mathrm{Cc}_{55}\mathrm{Ct}_{55})}(60\gamma_{13}\mathrm{Cb}_{55}\mathrm{Cc}_{55}y_3^3 + \gamma_{13}y_3(337\mathrm{Cb}_{55}\mathrm{Cc}_{55}h^2 - 188\mathrm{Cb}_{55}\mathrm{Ct}_{55}h^2 - 68\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^2) \\ &+ \gamma_{13}(-240\mathrm{Cb}_{55}\mathrm{Cc}_{55}h^3 + 195\mathrm{Cb}_{55}\mathrm{Ct}_{55}h^3 + 45\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^3)) \end{aligned}$$

$$\begin{aligned} \omega_1^c = &-\frac{1}{4h^2(\mathrm{Cb}_{55}(17\mathrm{Cc}_{55} + 47\mathrm{Ct}_{55}) + 17\mathrm{Cc}_{55}\mathrm{Ct}_{55})}(60\gamma_{13}\mathrm{Cb}_{55}\mathrm{Ct}_{55}y_3^3 - \gamma_{13}y_3(68\mathrm{Cb}_{55}\mathrm{Cc}_{55}h^2 - 217\mathrm{Cb}_{55}\mathrm{Ct}_{55}h^2 + 68\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^2) \\ &- \gamma_{13}(45\mathrm{Cb}_{55}\mathrm{Cc}_{55}h^3 - 45\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^3)) \end{aligned}$$

$$\begin{aligned} \omega_1^b = &-\frac{1}{4h^2(\mathrm{Cb}_{55}(17\mathrm{Cc}_{55} + 47\mathrm{Ct}_{55}) + 17\mathrm{Cc}_{55}\mathrm{Ct}_{55})}(60\gamma_{13}\mathrm{Cc}_{55}\mathrm{Ct}_{55}y_3^3 - \gamma_{13}y_3(68\mathrm{Cb}_{55}\mathrm{Cc}_{55}h^2 + 188\mathrm{Cb}_{55}\mathrm{Ct}_{55}h^2 - 337\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^2) \\ &- \gamma_{13}(45\mathrm{Cb}_{55}\mathrm{Cc}_{55}h^3 + 195\mathrm{Cb}_{55}\mathrm{Ct}_{55}h^3 - 240\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^3)) \end{aligned}$$

$$\begin{aligned} \omega_2^t = &-\frac{1}{4h^2(\mathrm{Cb}_{44}(17\mathrm{Cc}_{44} + 47\mathrm{Ct}_{44}) + 17\mathrm{Cc}_{44}\mathrm{Ct}_{44})}(60\gamma_{23}\mathrm{Cb}_{44}\mathrm{Cc}_{44}y_3^3 + \gamma_{23}y_3(337\mathrm{Cb}_{44}\mathrm{Cc}_{44}h^2 - 188\mathrm{Cb}_{44}\mathrm{Ct}_{44}h^2 - 68\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^2) \\ &+ \gamma_{23}(-240\mathrm{Cb}_{44}\mathrm{Cc}_{44}h^3 + 195\mathrm{Cb}_{44}\mathrm{Ct}_{44}h^3 + 45\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^3)) \end{aligned}$$

$$\begin{aligned} \omega_2^c = &-\frac{1}{4h^2(\mathrm{Cb}_{44}(17\mathrm{Cc}_{44} + 47\mathrm{Ct}_{44}) + 17\mathrm{Cc}_{44}\mathrm{Ct}_{44})}(60\gamma_{23}\mathrm{Cb}_{44}\mathrm{Ct}_{44}y_3^3 - \gamma_{23}y_3(68\mathrm{Cb}_{44}\mathrm{Cc}_{44}h^2 - 217\mathrm{Cb}_{44}\mathrm{Ct}_{44}h^2 + 68\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^2) \\ &- \gamma_{23}(45\mathrm{Cb}_{44}\mathrm{Cc}_{44}h^3 - 45\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^3)) \end{aligned}$$

$$\omega_2^b = -\frac{1}{4h^2(\mathrm{Cb}_{44}(17\mathrm{Cc}_{44}+47\mathrm{Ct}_{44})+17\mathrm{Cc}_{44}\mathrm{Ct}_{44})}(60\gamma_{23}\mathrm{Cc}_{44}\mathrm{Ct}_{44}y_3^3 - \gamma_{23}y_3(68\mathrm{Cb}_{44}\mathrm{Cc}_{44}h^2 + 188\mathrm{Cb}_{44}\mathrm{Ct}_{44}h^2 - 337\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^2)$$
$$- \gamma_{23}(45\mathrm{Cb}_{44}\mathrm{Cc}_{44}h^3 + 195\mathrm{Cb}_{44}\mathrm{Ct}_{44}h^3 - 240\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^3))$$

$$\omega_3^t = \frac{1}{864\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h}\mathrm{Cb}_{33}(\mathrm{Cc}_{33}(\mathrm{Cb}_{23}(h\kappa_{22}(167h^2 - 108hy_3 - 180y_3^2) + \epsilon_{22}(394h^2 - 648hy_3 + 72y_3^2)) + 12\mathrm{Ct}_{13}h(\kappa_{11}(19h^2 - 36y_3^2)$$
$$+ 24\epsilon_{11}(2h - 3y_3)) + \mathrm{Ct}_{23}(h\kappa_{22}(61h^2 + 108hy_3 - 252y_3^2) + 2\epsilon_{22}(91h^2 - 108hy_3 - 36y_3^2))) - 8\mathrm{Ct}_{33}h^2(-7\mathrm{Cb}_{23}(h\kappa_{22} + 2\epsilon_{22}) + 3\mathrm{Cc}_{23}h\kappa_{22}$$
$$+ 3\mathrm{Cc}_{13}(h\kappa_{11} + 18\epsilon_{11}) + 54\mathrm{Cc}_{23}\epsilon_{22} + 7\mathrm{Ct}_{23}h\kappa_{22} + 14\mathrm{Ct}_{23}\epsilon_{22})) + 8\mathrm{Cc}_{33}\mathrm{Ct}_{33}h^2(-3\mathrm{Cb}_{13}(6\epsilon_{11} - 5h\kappa_{11}) - \mathrm{Ct}_{23}(h\kappa_{22} + 2\epsilon_{22}))$$

$$\omega_3^c = \frac{1}{864\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h}8\mathrm{Cc}_{33}\mathrm{Ct}_{33}h^2(-3\mathrm{Cb}_{13}(6\epsilon_{11} - 5h\kappa_{11}) - 16\mathrm{Cb}_{23}(\epsilon_{22} - h\kappa_{22}) - \mathrm{Ct}_{23}(h\kappa_{22} + 2\epsilon_{22})) + \mathrm{Cb}_{33}(4\mathrm{Cc}_{33}h^2(\mathrm{Cb}_{23}(17h\kappa_{22} + 22\epsilon_{22})$$
$$+ 6\mathrm{Ct}_{13}(5h\kappa_{11} + 6\epsilon_{11}) + \mathrm{Ct}_{23}(13h\kappa_{22} + 14\epsilon_{22})) + \mathrm{Ct}_{33}(-\mathrm{Cb}_{23}(7h^2 - 108hy_3 - 36y_3^2)(h\kappa_{22} + 2\epsilon_{22}) + 84\mathrm{Cc}_{23}h^3\kappa_{22}$$
$$+ 12\mathrm{Cc}_{13}h(7h^2\kappa_{11} - 36\kappa_{11}y_3^2 - 72y_3\epsilon_{11}) - 432\mathrm{Cc}_{23}h\kappa_{22}y_3^2 - 864\mathrm{Cc}_{23}hy_3\epsilon_{22} + 7\mathrm{Ct}_{23}h^3\kappa_{22} - 108\mathrm{Ct}_{23}h^2\kappa_{22}y_3 + 14\mathrm{Ct}_{23}h^2\epsilon_{22} - 36\mathrm{Ct}_{23}h\kappa_{22}y_3^2$$
$$- 216\mathrm{Ct}_{23}hy_3\epsilon_{22} - 72\mathrm{Ct}_{23}y_3^2\epsilon_{22}))$$

$$\omega_3^b = \frac{1}{864\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h}\mathrm{Cc}_{33}\mathrm{Ct}_{33}(12\mathrm{Cb}_{13}h(\kappa_{11}(19h^2 - 36y_3^2) - 24\epsilon_{11}(2h + 3y_3)) + \mathrm{Cb}_{23}(h\kappa_{22}(281h^2 + 108hy_3$$
$$- 396y_3^2) + \epsilon_{22}(-470h^2 - 648hy_3 + 72y_3^2)) - \mathrm{Ct}_{23}(53h^2 + 108hy_3 + 36y_3^2)(h\kappa_{22} + 2\epsilon_{22})) + 4\mathrm{Cb}_{33}h^2(\mathrm{Ct}_{33}(-13\mathrm{Cb}_{23}(h\kappa_{22} + 2\epsilon_{22}) - 6\mathrm{Cc}_{23}h\kappa_{22}$$
$$+ 6\mathrm{Cc}_{13}(18\epsilon_{11} - h\kappa_{11}) + 108\mathrm{Cc}_{23}\epsilon_{22} + 13\mathrm{Ct}_{23}h\kappa_{22} + 26\mathrm{Ct}_{23}\epsilon_{22}) + \mathrm{Cc}_{33}(\mathrm{Cb}_{23}(17h\kappa_{22} + 22\epsilon_{22}) + 6\mathrm{Ct}_{13}(5h\kappa_{11} + 6\epsilon_{11}) + \mathrm{Ct}_{23}(13h\kappa_{22} + 14\epsilon_{22})))$$

Zeroth order stiffness coefficients:

$$A_{11} = \frac{1}{108\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}}h(\mathrm{Cb}_{13}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 26\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - \mathrm{Cb}_{13}(27\mathrm{Cb}_{33}(\mathrm{Cc}_{13} - \mathrm{Cc}_{31})\mathrm{Ct}_{33} + 2\mathrm{Cb}_{31}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 28\mathrm{Cc}_{33}\mathrm{Ct}_{33}))$$
$$+ 27\mathrm{Cb}_{31}\mathrm{Cb}_{33}(\mathrm{Cc}_{13} - \mathrm{Cc}_{31})\mathrm{Ct}_{33} + \mathrm{Cb}_{31}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 26\mathrm{Cc}_{33}\mathrm{Ct}_{33}) + 108\mathrm{Cb}_{33}(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cb}_{11} + \mathrm{Cc}_{11} + \mathrm{Ct}_{11}) - \mathrm{Ct}_{13}^2) - \mathrm{Cc}_{13}\mathrm{Cc}_{31}\mathrm{Ct}_{33}))$$

$$A_{12} = \frac{1}{216\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}}h(-216\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23} - 52\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cc}_{33}\mathrm{Ct}_{33} + 216\mathrm{Cb}_{12}\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33} + 216\mathrm{Cb}_{33}\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}$$
$$+ 216\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33} - 216\mathrm{Cb}_{33}\mathrm{Cc}_{13}\mathrm{Cc}_{23}\mathrm{Ct}_{33} - 2\mathrm{Cb}_{13}\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 28\mathrm{Cc}_{33}\mathrm{Ct}_{33}) + \mathrm{Cb}_{13}(2\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33})$$
$$- 26\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - 2\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 28\mathrm{Cc}_{33}\mathrm{Ct}_{33})) + 2\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cb}_{33}\mathrm{Cc}_{33} + 14\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cb}_{33}\mathrm{Ct}_{33})$$

$$B_{11} = -\frac{1}{216\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}}h^2(\mathrm{Cb}_{13}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 53\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - 2\mathrm{Cb}_{13}((\mathrm{Cc}_{13} - \mathrm{Cc}_{31})(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + \mathrm{Cc}_{33}\mathrm{Ct}_{33})$$
$$+ \mathrm{Cb}_{31}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 55\mathrm{Cc}_{33}\mathrm{Ct}_{33})) + 216\mathrm{Cb}_{33}\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cb}_{11} - \mathrm{Ct}_{11}) + \mathrm{Ct}_{13}^2) + \mathrm{Cb}_{31}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 53\mathrm{Cc}_{33}\mathrm{Ct}_{33})$$
$$+ 2\mathrm{Cb}_{31}(\mathrm{Cc}_{13} - \mathrm{Cc}_{31})(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + \mathrm{Cc}_{33}\mathrm{Ct}_{33}))$$

$$B_{12} = \frac{1}{432\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}}h^2(-432\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23} + 106\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cc}_{33}\mathrm{Ct}_{33} - 432\mathrm{Cb}_{12}\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33} + 432\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}$$
$$+ 2\mathrm{Cb}_{13}\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 55\mathrm{Cc}_{33}\mathrm{Ct}_{33}) + \mathrm{Cb}_{13}(2\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 55\mathrm{Cc}_{33}\mathrm{Ct}_{33})$$
$$- 2\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 53\mathrm{Cc}_{33}\mathrm{Ct}_{33})) - 2\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cb}_{33}\mathrm{Cc}_{33} - 14\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cb}_{33}\mathrm{Ct}_{33}))$$

$$A_{22} = \frac{1}{108\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}}h(\mathrm{Cb}_{23}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 26\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - \mathrm{Cb}_{23}(27\mathrm{Cb}_{33}(\mathrm{Cc}_{23} - \mathrm{Cc}_{32})\mathrm{Ct}_{33} + 2\mathrm{Cb}_{32}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33})$$
$$+ 28\mathrm{Cc}_{33}\mathrm{Ct}_{33})) + 27\mathrm{Cb}_{32}\mathrm{Cb}_{33}(\mathrm{Cc}_{23} - \mathrm{Cc}_{32})\mathrm{Ct}_{33} + \mathrm{Cb}_{32}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 26\mathrm{Cc}_{33}\mathrm{Ct}_{33})$$
$$+ 108\mathrm{Cb}_{33}(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cb}_{22} + \mathrm{Cc}_{22} + \mathrm{Ct}_{22}) - \mathrm{Ct}_{23}^2) - \mathrm{Cc}_{23}\mathrm{Cc}_{32}\mathrm{Ct}_{33}))$$

$$B_{22} = -\frac{1}{216\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}}h^2(\mathrm{Cb}_{23}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 53\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - 2\mathrm{Cb}_{23}((\mathrm{Cc}_{23} - \mathrm{Cc}_{32})(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + \mathrm{Cc}_{33}\mathrm{Ct}_{33})$$
$$+ \mathrm{Cb}_{32}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 55\mathrm{Cc}_{33}\mathrm{Ct}_{33})) + 216\mathrm{Cb}_{33}\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cb}_{22} - \mathrm{Ct}_{22}) + \mathrm{Ct}_{23}^2) + \mathrm{Cb}_{32}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 53\mathrm{Cc}_{33}\mathrm{Ct}_{33})$$
$$+ 2\mathrm{Cb}_{32}(\mathrm{Cc}_{23} - \mathrm{Cc}_{32})(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + \mathrm{Cc}_{33}\mathrm{Ct}_{33})))$$

$$S_{11} = \frac{405\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h}{\mathrm{Cb}_{55}(34\mathrm{Cc}_{55} + 94\mathrm{Ct}_{55}) + 34\mathrm{Cc}_{55}\mathrm{Ct}_{55}},\ S_{22} = \frac{405\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h}{\mathrm{Cb}_{44}(34\mathrm{Cc}_{44} + 94\mathrm{Ct}_{44}) + 34\mathrm{Cc}_{44}\mathrm{Ct}_{44}},\ A_{33} = h(2\mathrm{Cb}_{66} + \mathrm{Ct}_{66}),\ B_{33} = h^2(\mathrm{Ct}_{66} - \mathrm{Cb}_{66})$$

$$D_{11} = \frac{1}{432\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}}h^3(2\mathrm{Cb}_{13}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 116\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - 2\mathrm{Cb}_{13}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33})$$
$$+ 118\mathrm{Cc}_{33}\mathrm{Ct}_{33}) + 4\mathrm{Cb}_{33}(9\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(13\mathrm{Cb}_{11} + \mathrm{Cc}_{11} + 13\mathrm{Ct}_{11}) - 13\mathrm{Ct}_{13}^2) + 2\mathrm{Cc}_{13}^2(\mathrm{Cc}_{33} - 2\mathrm{Ct}_{33}) - \mathrm{Cc}_{13}^2(2\mathrm{Cc}_{33} + 5\mathrm{Ct}_{33})))$$

$$
\begin{aligned}
D_{12} = {} & \frac{1}{864\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(-936\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23} - 232\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cc}_{33}\mathrm{Ct}_{33} + 936\mathrm{Cb}_{12}\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33} + 72\mathrm{Cb}_{33}\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33} \\
& + 936\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33} - 72\mathrm{Cb}_{33}\mathrm{Cc}_{13}\mathrm{Cc}_{23}\mathrm{Ct}_{33} - 2\mathrm{Cb}_{13}\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 118\mathrm{Cc}_{33}\mathrm{Ct}_{33}) + \mathrm{Cb}_{13}(2\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) \\
& - 116\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - 2\mathrm{Cb}_{23}(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 118\mathrm{Cc}_{33}\mathrm{Ct}_{33})) + 2\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cb}_{33}\mathrm{Cc}_{33} + 14\mathrm{Cb}_{13}\mathrm{Cb}_{23}\mathrm{Cb}_{33}\mathrm{Ct}_{33})
\end{aligned}
$$

$$
\begin{aligned}
D_{22} = {} & \frac{1}{432\mathrm{Cb}_{33}\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(2\mathrm{Cb}_{23}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) - 116\mathrm{Cc}_{33}\mathrm{Ct}_{33}) - 2\mathrm{Cb}_{23}^2(\mathrm{Cb}_{33}(\mathrm{Cc}_{33} + 7\mathrm{Ct}_{33}) + 118\mathrm{Cc}_{33}\mathrm{Ct}_{33}) \\
& + 4\mathrm{Cb}_{33}(9\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(13\mathrm{Cb}_{22} + \mathrm{Cc}_{22} + 13\mathrm{Ct}_{22}) - 13\mathrm{Ct}_{23}^2) + 2\mathrm{Cc}_{23}^2(\mathrm{Cc}_{33} - 2\mathrm{Ct}_{33}) - \mathrm{Cc}_{23}^2(2\mathrm{Cc}_{33} + 5\mathrm{Ct}_{33})))
\end{aligned}
$$

$$
D_{33} = \frac{1}{12} h^3(14\mathrm{Cb}_{66} + 13\mathrm{Ct}_{66}),\;\; A_{13} = B_{13} = A_{23} = B_{23} = A_{31} = A_{32} = B_{31} = B_{32} = D_{13} = D_{23} = D_{31} = D_{32} = S_{12} = S_{21} = 0
$$

The first order approximation is carried out by taking next higher order contributions to the energy functional in an asymptotic sense. The first order stiffness coefficients are illustrated using the equation below:

$$
[\overline{Q}] = [Q] + \Big\{ X \Big\} \Big\{ \epsilon \quad \kappa \quad \gamma \Big\} + [Z] \begin{Bmatrix} \epsilon \\ \kappa \\ \gamma \end{Bmatrix} \Big\{ \epsilon \quad \kappa \quad \gamma \Big\} \tag{11}
$$

where $\overline{Q}$ is the first order 2-D stiffness matrix, which consists of sub matrices $\overline{A}, \overline{B}, \overline{D}$ and $\overline{S}$. These sub matrices $\overline{A}, \overline{B}, \overline{D}$ are of size 3×3 and $\overline{S}$ is of size 2×2; the elements in these sub matrices represents the extensional, coupling, bending and shear stiffnesses respectively. The matrix Q is of size 8×8 represents the constant part of $\overline{Q}$. The vector X is of size 8×1 and the matrix Z is of size 8×8 represent the linear and quadratic part of $\overline{Q}$ respectively. In first order approximation, the perturbed warping functions are determined from the minimization process of first order total potential energy. The first order warping functions are representing for top surface as v_1^t, v_2^t and v_3^t; for core surface as v_1^c, v_2^c and v_3^c; for bottom surface as v_1^b, v_2^b and v_3^b. The analytical expressions of these first order warping functions are expressed below:

$$
\begin{aligned}
v_1^t = {} & -\frac{1}{4h^2(\mathrm{Cb}_{55}^2(19\mathrm{Cc}_{55} + 29\mathrm{Ct}_{55}) + \mathrm{Cb}_{55}\mathrm{Ct}_{55}(81\mathrm{Cc}_{55} + 112\mathrm{Ct}_{55}) + 2\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2)}(\gamma_{13}y_3^3(60\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55} + 120\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}) \\
& - \gamma_{13}y_3^2(180\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h - 180\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}h) - \gamma_{13}y_3(-621\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^2 - 194\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}h^2 + 448\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2h^2 \\
& + 116\mathrm{Cb}_{55}^2\mathrm{Ct}_{55}h^2 + 8\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2h^2) - \gamma_{13}(375\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^3 + 215\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}h^3 - 455\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2h^3 - 130\mathrm{Cb}_{55}^2\mathrm{Ct}_{55}h^3 - 5\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2h^3))
\end{aligned}
$$

$$
\begin{aligned}
v_1^c = {} & -\frac{1}{4h^2(\mathrm{Cb}_{55}^2(19\mathrm{Cc}_{55} + 29\mathrm{Ct}_{55}) + \mathrm{Cb}_{55}\mathrm{Ct}_{55}(81\mathrm{Cc}_{55} + 112\mathrm{Ct}_{55}) + 2\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2)}(\gamma_{13}y_3^3(120\mathrm{Cb}_{55}^2\mathrm{Ct}_{55} + 60\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2) \\
& - \gamma_{13}y_3^2(180\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2h - 180\mathrm{Cb}_{55}^2\mathrm{Ct}_{55}h) - \gamma_{13}y_3(324\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^2 + 76\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}h^2 - 497\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2h^2 \\
& - 154\mathrm{Cb}_{55}^2\mathrm{Ct}_{55}h^2 + 8\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2h^2) - \gamma_{13}(-45\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^3 + 50\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}h^3 - 35\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2h^3 + 35\mathrm{Cb}_{55}^2\mathrm{Ct}_{55}h^3 - 5\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2h^3))
\end{aligned}
$$

$$
\begin{aligned}
v_1^b = {} & -\frac{1}{4h^2(\mathrm{Cb}_{55}^2(19\mathrm{Cc}_{55} + 29\mathrm{Ct}_{55}) + \mathrm{Cb}_{55}\mathrm{Ct}_{55}(81\mathrm{Cc}_{55} + 112\mathrm{Ct}_{55}) + 2\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2)}(\gamma_{13}y_3^3(120\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55} + 60\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2) \\
& - \gamma_{13}y_3^2(180\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2h - 180\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h) - \gamma_{13}y_3(-1026\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^2 + 76\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}h^2 + 448\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2h^2 + 116\mathrm{Cb}_{55}^2\mathrm{Ct}_{55}h^2 \\
& + 143\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2h^2) - \gamma_{13}(-660\mathrm{Cb}_{55}\mathrm{Cc}_{55}\mathrm{Ct}_{55}h^3 + 50\mathrm{Cb}_{55}^2\mathrm{Cc}_{55}h^3 + 475\mathrm{Cb}_{55}\mathrm{Ct}_{55}^2h^3 + 110\mathrm{Cb}_{55}^2\mathrm{Ct}_{55}h^3 + 25\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2h^3))
\end{aligned}
$$

$$
\begin{aligned}
v_2^t = &-\frac{1}{4h^2(\mathrm{Cb}_{44}^2(19\mathrm{Cc}_{44}+29\mathrm{Ct}_{44})+\mathrm{Cb}_{44}\mathrm{Ct}_{44}(81\mathrm{Cc}_{44}+112\mathrm{Ct}_{44})+2\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2)}(\gamma_{23}y_3^3(60\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}+120\mathrm{Cb}_{44}^2\mathrm{Cc}_{44})\\
&-\gamma_{23}y_3^2(180\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h-180\mathrm{Cb}_{44}^2\mathrm{Cc}_{44}h)-\gamma_{23}y_3(-621\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^2-194\mathrm{Cb}_{44}^2\mathrm{Cc}_{44}h^2+448\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2h^2+116\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}h^2\\
&+8\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2h^2)-\gamma_{23}(375\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^3+215\mathrm{Cb}_{44}^2\mathrm{Cc}_{44}h^3-455\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2h^3-130\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}h^3-5\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2h^3))
\end{aligned}
$$

$$
\begin{aligned}
v_2^c = &-\frac{1}{4h^2(\mathrm{Cb}_{44}^2(19\mathrm{Cc}_{44}+29\mathrm{Ct}_{44})+\mathrm{Cb}_{44}\mathrm{Ct}_{44}(81\mathrm{Cc}_{44}+112\mathrm{Ct}_{44})+2\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2)}(\gamma_{23}y_3^3(120\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}+60\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2)\\
&-\gamma_{23}y_3^2(180\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2h-180\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}h)-\gamma_{23}y_3(324\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^2+76\mathrm{Cb}_{44}^2\mathrm{Cc}_{44}h^2-497\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2h^2-154\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}h^2\\
&+8\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2h^2)-\gamma_{23}(-45\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^3+50\mathrm{Cb}_{44}^2\mathrm{Cc}_{44}h^3-35\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2h^3+35\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}h^3-5\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2h^3))
\end{aligned}
$$

$$
\begin{aligned}
v_2^b = &-\frac{1}{4h^2(\mathrm{Cb}_{44}^2(19\mathrm{Cc}_{44}+29\mathrm{Ct}_{44})+\mathrm{Cb}_{44}\mathrm{Ct}_{44}(81\mathrm{Cc}_{44}+112\mathrm{Ct}_{44})+2\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2)}(\gamma_{23}y_3^3(120\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}+60\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2)\\
&-\gamma_{23}y_3^2(180\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2h-180\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h)-\gamma_{23}y_3(-1026\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^2+76\mathrm{Cb}_{44}^2\mathrm{Cc}_{44}h^2+448\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2h^2+116\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}h^2\\
&+143\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2h^2)-\gamma_{23}(-660\mathrm{Cb}_{44}\mathrm{Cc}_{44}\mathrm{Ct}_{44}h^3+50\mathrm{Cb}_{44}^2\mathrm{Cc}_{44}h^3+475\mathrm{Cb}_{44}\mathrm{Ct}_{44}^2h^3+110\mathrm{Cb}_{44}^2\mathrm{Ct}_{44}h^3+25\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2h^3))
\end{aligned}
$$

$$
\begin{aligned}
v_3^t = &\frac{1}{15552\mathrm{Cc}_{33}^2\mathrm{Ct}_{33}^2}\kappa_{11}(2\mathrm{Cc}_{33}(52\mathrm{Ct}_{33}(6\epsilon_{11}-5h\kappa_{11})h^2+\mathrm{Cc}_{33}(h(449h^2-540y_3h+180y_3^2)\kappa_{11}-6(97h^2-108y_3h+36y_3^2)\epsilon_{11}))\mathrm{Cb}_{13}^2\\
&-(3(3\mathrm{Ct}_{33}((47h\kappa_{11}+4)h^2+108y_3(h\kappa_{11}-4)h-36y_3^2(h\kappa_{11}-4))-8(\mathrm{Ct}_{13}(45(h-2y_3)\kappa_{11}h^2+(36y_3^2-29h^2)\epsilon_{11})))\\
&+2\mathrm{Ct}_{33}(234\mathrm{Ct}_{33}(h\kappa_{11}-4)h^2+\mathrm{Cc}_{13}(25h^2-108y_3h+36y_3^2)(h\kappa_{11}-18\epsilon_{11})+6(2h^2\mathrm{Ct}_{13}(45h\kappa_{11}-2\epsilon_{11})-3\mathrm{Cc}_{23}(25h^2)\epsilon_{22}))\\
&-104h^2\mathrm{Ct}_{33}^2(\mathrm{Cc}_{13}(h\kappa_{11}-18\epsilon_{11})-18\mathrm{Cc}_{23}\epsilon_{22}))\mathrm{Cb}_{13}+8h^2\mathrm{Ct}_{33}^2(9(4\epsilon_{11}-9h\kappa_{11})\mathrm{Cc}_{13}^2+2(18\mathrm{Cc}_{23}\epsilon_{22}+7\mathrm{Ct}_{13}(18\epsilon_{11}+h\kappa_{11}))\mathrm{Cc}_{13}\\
&+252\mathrm{Cc}_{23}\mathrm{Ct}_{13}\epsilon_{22})-2\mathrm{Cc}_{33}\mathrm{Ct}_{33}(36(61h^2-108y_3h+36y_3^2)\epsilon_{11}\mathrm{Cc}_{13}^2+(54\mathrm{Ct}_{33}(3h\kappa_{11}+4)h^2+36\mathrm{Cc}_{23}(61h^2-108y_3h+36y_3^2)\epsilon_{22}\\
&+\mathrm{Ct}_{13}(83h^2-108y_3h-36y_3^2)(18\epsilon_{11}+h\kappa_{11}))\mathrm{Cc}_{13}+2(162(\mathrm{Cc}_{11}+12\mathrm{Ct}_{11})\mathrm{Ct}_{33}\kappa_{11}h^3+28\mathrm{Ct}_{13}^2(6\epsilon_{11}+5h\kappa_{11})h^2-9\mathrm{Ct}_{13}(13\mathrm{Ct}_{33}(h\kappa_{11}-4)h^2\\
&+\mathrm{Cc}_{23}(-83h^2+108y_3h+36y_3^2))))+\mathrm{Cc}_{33}^2((12(173h^2-756y_3h+612y_3^2)\epsilon_{11}+2(415h^3-1512y_3h^2-180y_3^2h+1296y_3^3)\kappa_{11})\mathrm{Ct}_{13}^2\\
&+9(24\mathrm{Ct}_{23}(5h^2-36y_3h+36y_3^2)\epsilon_{22}+\mathrm{Ct}_{33}(-288\kappa_{11}y_3^3-36(h\kappa_{11}+20)y_3^2+108h(h\kappa_{11}-4)y_3+h^2(83h\kappa_{11}+700)))\mathrm{Ct}_{13}\\
&+648\mathrm{Ct}_{11}\mathrm{Ct}_{33}(13h^3-27y_3h^2+4y_3^3)\kappa_{11}))
\end{aligned}
$$

$$
\begin{aligned}
v_3^c = &\frac{1}{15552h\mathrm{Cc}_{33}^2\mathrm{Ct}_{33}^2}2h\mathrm{Cc}_{33}\kappa_{11}(8h^2\mathrm{Cc}_{33}(28h\kappa_{11}-39\epsilon_{11})-\mathrm{Ct}_{33}(7h^2+108y_3h-36y_3^2)(5h\kappa_{11}-6\epsilon_{11}))\mathrm{Cb}_{13}^2-(12h^2(8h(5\mathrm{Ct}_{13}\epsilon_{11}+9\mathrm{Ct}_{23}\epsilon_{22})\kappa_{11}\\
&+3\mathrm{Ct}_{33}(20\epsilon_{11}^2+(40-44h\kappa_{11})\epsilon_{11}+h\kappa_{11}(23h\kappa_{11}-44)))\mathrm{Cc}_{33}^2+\mathrm{Ct}_{33}(9\mathrm{Ct}_{33}(7h^2+108y_3h-36y_3^2)(4\epsilon_{11}^2+(8-4h\kappa_{11})\epsilon_{11}+h\kappa_{11}(h\kappa_{11}-4))\\
&+8h\kappa_{11}(3(30\mathrm{Cc}_{23}\epsilon_{22}h^2+\mathrm{Ct}_{13}(90y_3\kappa_{11}h^2+(7h^2-36y_3^2)\epsilon_{11}))-5h^2\mathrm{Cc}_{13}(h\kappa_{11}-18\epsilon_{11})))\mathrm{Cc}_{33}-2h\mathrm{Ct}_{33}^2(7h^2+108y_3h-36y_3^2)\\
&\kappa_{11}(\mathrm{Cc}_{13}(h\kappa_{11}-18\epsilon_{11})-18\mathrm{Cc}_{23}\epsilon_{22}))\mathrm{Cb}_{13}+2h\mathrm{Ct}_{33}^2\kappa_{11}(-36(2(7h^2-36y_3^2)\epsilon_{11}+9y_3(3h^2-4y_3^2)\kappa_{11})\mathrm{Cc}_{13}^2+(72\mathrm{Cc}_{23}(36y_3^2-7h^2)\epsilon_{22}\\
&-\mathrm{Ct}_{13}(7h^2-108y_3h-36y_3^2)(18\epsilon_{11}+h\kappa_{11}))\mathrm{Cc}_{13}+18\mathrm{Cc}_{23}\mathrm{Ct}_{13}(-7h^2+108y_3h+36y_3^2)\epsilon_{22})-4h^2\mathrm{Cc}_{33}^2(4h\kappa_{11}(39\epsilon_{11}+28h\kappa_{11})\mathrm{Ct}_{13}^2\\
&-9(\mathrm{Ct}_{33}(20\epsilon_{11}^2+(76h\kappa_{11}+40)\epsilon_{11}+h\kappa_{11}(23h\kappa_{11}+76))-24h\mathrm{Ct}_{23}\epsilon_{22}\kappa_{11})\mathrm{Ct}_{13}+288h\mathrm{Ct}_{33}(\mathrm{Cc}_{11}\epsilon_{11}-\mathrm{Ct}_{11}\epsilon_{11}+(\mathrm{Cc}_{12}-\mathrm{Ct}_{12})\epsilon_{22})\kappa_{11})\\
&+\mathrm{Cc}_{33}\mathrm{Ct}_{33}(2h(7h^2-108y_3h-36y_3^2)\kappa_{11}(6\epsilon_{11}+5h\kappa_{11})\mathrm{Ct}_{13}^2+(9\mathrm{Ct}_{33}(7h^2+108y_3h-36y_3^2)(4\epsilon_{11}^2+(8-4h\kappa_{11})\epsilon_{11}+h\kappa_{11}(h\kappa_{11}-4))\\
&-40h^3\kappa_{11}(18\mathrm{Cc}_{23}\epsilon_{22}+\mathrm{Cc}_{13}(18\epsilon_{11}+h\kappa_{11})))\mathrm{Ct}_{13}-72h(16h^2\epsilon_{11}\kappa_{11}\mathrm{Cc}_{13}^2+(16\mathrm{Cc}_{23}\epsilon_{22}\kappa_{11}h^2+3\mathrm{Ct}_{33}(12\kappa_{11}^2y_3^3\\
&+36(\epsilon_{11}+1)\kappa_{11}y_3^2+36\epsilon_{11}(\epsilon_{11}+2)y_3-7h^2(\epsilon_{11}+1)\kappa_{11}))\mathrm{Cc}_{13}+\mathrm{Ct}_{33}(108\mathrm{Cc}_{23}y_3\epsilon_{22}(\epsilon_{22}+2)+\kappa_{11}(\mathrm{Cc}_{11}(2(7h^2-36y_3^2)\epsilon_{11}\\
&+9y_3(3h^2-4y_3^2)\kappa_{11})+2((\mathrm{Cc}_{12}-\mathrm{Ct}_{12})(7h^2-36y_3^2)\epsilon_{22}))))))
\end{aligned}
$$

$$
\begin{aligned}
v_3^b = &\frac{1}{15552h\mathrm{Cc}_{33}^2\mathrm{Ct}_{33}^2}8\mathrm{Ct}_{33}^2\kappa_{11}(9(4\epsilon_{11}+9h\kappa_{11})\mathrm{Cc}_{13}^2+(36\mathrm{Cc}_{23}\epsilon_{22}-13\mathrm{Ct}_{13}(18\epsilon_{11}+h\kappa_{11}))\mathrm{Cc}_{13}-234\mathrm{Cc}_{23}\mathrm{Ct}_{13}\epsilon_{22})h^3\\
&+2\mathrm{Cb}_{13}^2\mathrm{Cc}_{33}\kappa_{11}(56\mathrm{Ct}_{33}(5h\kappa_{11}-6\epsilon_{11})h^2+\mathrm{Cc}_{33}(6(173h^2+756y_3h+612y_3^2)\epsilon_{11}+(-415h^3-1512y_3h^2+180y_3^2h+1296y_3^3)\kappa_{11}))h\\
&+2\mathrm{Cc}_{33}\mathrm{Ct}_{33}(-36(61h^2+108y_3h+36y_3^2)\epsilon_{11}\kappa_{11}\mathrm{Cc}_{13}^2+(\kappa_{11}(\mathrm{Ct}_{13}(25h^2+108y_3h+36y_3^2)(18\epsilon_{11}+h\kappa_{11})-36\mathrm{Cc}_{23}(61h^2+108y_3h+36y_3^2)\epsilon_{22})\\
&+54h\mathrm{Ct}_{33}(36\epsilon_{11}^2+(72-4h\kappa_{11})\epsilon_{11}+h\kappa_{11}(3h\kappa_{11}-4)))\mathrm{Cc}_{13}+2(26h^2\kappa_{11}(6\epsilon_{11}+5h\kappa_{11})\mathrm{Ct}_{13}^2-9(14h\mathrm{Ct}_{33}(4\epsilon_{11}^2+(8-4h\kappa_{11})\epsilon_{11}+h\kappa_{11}(h\kappa_{11}-4))\\
&-\mathrm{Cc}_{23}(25h^2+108y_3h+36y_3^2)\epsilon_{22}\kappa_{11})\mathrm{Ct}_{13}+18h\mathrm{Ct}_{33}(54\mathrm{Cc}_{23}\epsilon_{22}(\epsilon_{22}+2)+h\kappa_{11}(\mathrm{Cc}_{11}(4\epsilon_{11}+9h\kappa_{11})-4((\mathrm{Ct}_{12}-\mathrm{Cc}_{12})\epsilon_{22}+\mathrm{Ct}_{11}(\epsilon_{11}-27h\kappa_{11}))))))h\\
&+\mathrm{Cb}_{13}(-112\mathrm{Ct}_{33}^2\kappa_{11}(\mathrm{Cc}_{13}(h\kappa_{11}-18\epsilon_{11})-18\mathrm{Cc}_{23}\epsilon_{22})h^3+2\mathrm{Cc}_{33}\mathrm{Ct}_{33}(252h\mathrm{Ct}_{33}(4\epsilon_{11}^2+(8-4h\kappa_{11})\epsilon_{11}+h\kappa_{11}(h\kappa_{11}-4))\\
&+\kappa_{11}(\mathrm{Cc}_{13}(83h^2+108y_3h-36y_3^2)(h\kappa_{11}-18\epsilon_{11})+6(2h^2\mathrm{Ct}_{13}(2\epsilon_{11}+45h\kappa_{11})-3\mathrm{Cc}_{23}(83h^2+108y_3h-36y_3^2)\epsilon_{22})))h\\
&-3\mathrm{Cc}_{33}^2(8h\kappa_{11}(\mathrm{Ct}_{13}(45(h+2y_3)\kappa_{11}h^2+(29h^2-36y_3^2)\epsilon_{11})-9\mathrm{Ct}_{23}(5h^2+36y_3h+36y_3^2)\epsilon_{22})+3\mathrm{Ct}_{33}(4(83h^2+108y_3h-36y_3^2)\epsilon_{11}^2\\
&+4((166-53h\kappa_{11})h^2+108y_3(h\kappa_{11}+2)h+36y_3^2(7h\kappa_{11}-2))\epsilon_{11}+h\kappa_{11})))))-\mathrm{Cc}_{33}^2(2h\kappa_{11}(6(97h^2+108y_3h+36y_3^2)\epsilon_{11}+h(449h^2+540y_3h+180y_3^2)\kappa_{11})\mathrm{Ct}_{13}^2\\
&-9(\mathrm{Ct}_{33}(-4(25h^2+108y_3h+36y_3^2)\epsilon_{11}^2+4((121h\kappa_{11}-50)h^2+108y_3(h\kappa_{11}-2)h+36y_3^2(h\kappa_{11}-2))\epsilon_{11})\mathrm{Ct}_{13}+72h\mathrm{Ct}_{33}(54\mathrm{Ct}_{23}(h+2y_3)\epsilon_{22}(\epsilon_{22}+2)\\
&+\kappa_{11}(\mathrm{Cc}_{11}(61h^2+36y_3^2)\epsilon_{11}+(\mathrm{Cc}_{12}-\mathrm{Ct}_{12})(61h^2+36y_3^2)\epsilon_{22}+\mathrm{Ct}_{11}\epsilon_{11}))))
\end{aligned}
$$

First order stiffness coefficients:

$$
\begin{aligned}
\overline{A_{11}} = {} & \frac{1}{96\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cc}_{11}(12\gamma_{12}^2 + 48\gamma_{13}^2 + 156h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 288h\kappa_{11} - 288h\kappa_{11}\epsilon_{11} + 144\epsilon_{11}^2 + 288\epsilon_{11}) + \mathrm{Cc}_{12}(12\gamma_{12}^2 \\
& + 48\gamma_{23}^2 + 52h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 96h\kappa_{22} - 96h\kappa_{22}\epsilon_{22} + 48\epsilon_{22}^2 + 96\epsilon_{22}) + 2(3\mathrm{Ct}_{66}(12\gamma_{12}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega) + \mathrm{Ct}_{11}(12\gamma_{12}^2 + 48\gamma_{13}^2 \\
& + 156h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 288h\kappa_{11} - 288h\kappa_{11}\epsilon_{11} + 144\epsilon_{11}^2 + 288\epsilon_{11}) + \mathrm{Ct}_{12}(12\gamma_{12}^2 + 48\gamma_{23}^2 + 52h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 96h\kappa_{22} \\
& - 96h\kappa_{22}\epsilon_{22} + 48\epsilon_{22}^2 + 96\epsilon_{22}))) - 2\mathrm{Ct}_{13}^2(12\gamma_{12}^2 + 48\gamma_{13}^2 + 156h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 288h\kappa_{11} - 288h\kappa_{11}\epsilon_{11} + 144\epsilon_{11}^2 + 288\epsilon_{11} + 96) \\
& - 2\mathrm{Ct}_{23}\mathrm{Ct}_{13}(12\gamma_{12}^2 + 48\gamma_{23}^2 + 52h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 96h\kappa_{22} - 96h\kappa_{22}\epsilon_{22} + 48\epsilon_{22}^2 + 96\epsilon_{22})) - \mathrm{Ct}_{33}(\mathrm{Cc}_{13}(\mathrm{Cc}_{23}(12\gamma_{12}^2 + 48\gamma_{23}^2 \\
& + 52h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 96h\kappa_{22} - 96h\kappa_{22}\epsilon_{22} + 48\epsilon_{22}^2 + 96\epsilon_{22}) + 192\mathrm{Ct}_{13}) + \mathrm{Cc}_{13}^2(12\gamma_{12}^2 + 48\gamma_{13}^2 + 156h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega \\
& - 288h\kappa_{11} - 288h\kappa_{11}\epsilon_{11} + 144\epsilon_{11}^2 + 288\epsilon_{11}) - 96\mathrm{Ct}_{13}^2))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{A_{12}} = {} & \frac{1}{48\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(4(\mathrm{Cc}_{12} + 2\mathrm{Ct}_{12})(h(\kappa_{11}(13h\kappa_{22} - 12) - 12\kappa_{22}) - 12\epsilon_{22}(h\kappa_{11} - 1) + 12\epsilon_{11}(-h\kappa_{22} + \epsilon_{22} + 1)) + \mathrm{Ct}_{66}(36\gamma_{12}^2 \\
& + 39h^2\omega^2 - 72\gamma_{12}h\omega)) - 8\mathrm{Ct}_{13}\mathrm{Ct}_{23}(13h^2\kappa_{11}\kappa_{22} - 12h\kappa_{11} - 12h\kappa_{22} - 12\epsilon_{22}(h\kappa_{11} - 1) + 12\epsilon_{11}(-h\kappa_{22} + \epsilon_{22} + 1) + 12)) \\
& - 4\mathrm{Ct}_{33}(\mathrm{Cc}_{23}(\mathrm{Cc}_{13}(h(\kappa_{11}(13h\kappa_{22} - 12) - 12\kappa_{22}) - 12\epsilon_{22}(h\kappa_{11} - 1) + 12\epsilon_{11}(-h\kappa_{22} + \epsilon_{22} + 1)) + 12\mathrm{Ct}_{13}) + 12\mathrm{Ct}_{23}(\mathrm{Cc}_{13} - \mathrm{Ct}_{13})))
\end{aligned}
$$

$$
\begin{aligned}
\overline{A_{13}} = {} & \frac{1}{48\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cc}_{11}(12\gamma_{12}(-h\kappa_{11} + \epsilon_{11} + 1) + h\omega(13h\kappa_{11} - 12\epsilon_{11} - 12)) + \mathrm{Cc}_{12}(12\gamma_{12}(-h\kappa_{11} + \epsilon_{11} + 1) + h\omega(13h\kappa_{11} - 12\epsilon_{11} \\
& - 12)) + 2(\mathrm{Ct}_{11}(12\gamma_{12}(-h\kappa_{11} + \epsilon_{11} + 1) + h\omega(13h\kappa_{11} - 12\epsilon_{11} - 12)) + \mathrm{Ct}_{12}(12\gamma_{12}(-h\kappa_{11} + \epsilon_{11} + 1) + h\omega(13h\kappa_{11} - 12\epsilon_{11} - 12)) \\
& + 3\mathrm{Ct}_{66}(12\gamma_{13}\gamma_{23} + 12\gamma_{12}(-h\kappa_{11} - h\kappa_{22} + \epsilon_{11} + \epsilon_{22} + 2) + h\omega(13h\kappa_{11} + 13h\kappa_{22} - 12\epsilon_{11} - 12\epsilon_{22} \\
& - 24)))) + \mathrm{Ct}_{13}^2(2h\omega(-13h\kappa_{11} + 12\epsilon_{11} + 12) - 24\gamma_{12}(-h\kappa_{11} + \epsilon_{11} + 1)) + 2\mathrm{Ct}_{23}\mathrm{Ct}_{13}(h\omega(-13h\kappa_{11} + 12\epsilon_{11} + 12) - 12\gamma_{12}(-h\kappa_{11} + \epsilon_{11} + 1))) \\
& - \mathrm{Cc}_{13}(\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(12\gamma_{12}(-h\kappa_{11} + \epsilon_{11} + 1) + h\omega(13h\kappa_{11} - 12\epsilon_{11} - 12)))
\end{aligned}
$$

$$
\begin{aligned}
\overline{A_{22}} = {} & \frac{1}{96\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cc}_{12}(12\gamma_{12}^2 + 48\gamma_{13}^2 + 52h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 96h\kappa_{11} - 96h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11}) + \mathrm{Cc}_{22}(12\gamma_{12}^2 + 48\gamma_{23}^2 \\
& + 156h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 288h\kappa_{22} - 288h\kappa_{22}\epsilon_{22} + 144\epsilon_{22}^2 + 288\epsilon_{22}) + 2(3\mathrm{Ct}_{66}(12\gamma_{12}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega) + \mathrm{Ct}_{12}(12\gamma_{12}^2 + 48\gamma_{13}^2 \\
& + 52h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 96h\kappa_{11} - 96h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11}) + \mathrm{Ct}_{22}(12\gamma_{12}^2 + 48\gamma_{23}^2 + 156h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 288h\kappa_{22} \\
& - 288h\kappa_{22}\epsilon_{22} + 144\epsilon_{22}^2 + 288\epsilon_{22}))) - 2\mathrm{Ct}_{23}^2(12\gamma_{12}^2 + 48\gamma_{23}^2 + 156h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 288h\kappa_{22} - 288h\kappa_{22}\epsilon_{22} + 144\epsilon_{22}^2 + 288\epsilon_{22} + 96) \\
& - 2\mathrm{Ct}_{13}\mathrm{Ct}_{23}(12\gamma_{12}^2 + 48\gamma_{13}^2 + 52h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 96h\kappa_{11} - 96h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11})) - \mathrm{Ct}_{33}(192\mathrm{Cc}_{23}\mathrm{Ct}_{23} + \mathrm{Cc}_{23}^2(12\gamma_{12}^2 + 48\gamma_{23}^2 \\
& + 156h^2\kappa_{22}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega - 288h\kappa_{22} - 288h\kappa_{22}\epsilon_{22} + 144\epsilon_{22}^2 + 288\epsilon_{22}) + \mathrm{Cc}_{13}\mathrm{Cc}_{23}(12\gamma_{12}^2 + 48\gamma_{13}^2 + 52h^2\kappa_{11}^2 + 13h^2\omega^2 - 24\gamma_{12}h\omega \\
& - 96h\kappa_{11} - 96h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11}) - 96\mathrm{Ct}_{23}^2))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{A_{23}} = {} & \frac{13}{8}\omega\mathrm{Ct}_{66}\kappa_{11}h^3 + \frac{13}{48}\omega\mathrm{Cc}_{12}\kappa_{22}h^3 + \frac{13}{48}\omega\mathrm{Cc}_{22}\kappa_{22}h^3 + \frac{13}{24}\omega\mathrm{Ct}_{12}\kappa_{22}h^3 + \frac{13}{24}\omega\mathrm{Ct}_{22}\kappa_{22}h^3 + \frac{13}{8}\omega\mathrm{Ct}_{66}\kappa_{22}h^3 - \frac{1}{48\mathrm{Cc}_{33}}13\omega\mathrm{Cc}_{23}^2\kappa_{22}h^3 \\
& - 13\omega\mathrm{Cc}_{13}\mathrm{Cc}_{23}\kappa_{22}h^3 - \frac{1}{24\mathrm{Ct}_{33}}(13\omega\mathrm{Ct}_{23}^2\kappa_{22}h^3 - 13\omega\mathrm{Ct}_{13}\mathrm{Ct}_{23}\kappa_{22}h^3) - \frac{1}{4}\omega\mathrm{Cc}_{12}h^2 - \frac{1}{4}\omega\mathrm{Cc}_{22}h^2 - \frac{1}{2}\omega\mathrm{Ct}_{12}h^2 - \frac{1}{2}\omega\mathrm{Ct}_{22}h^2 - 3\omega\mathrm{Ct}_{66}h^2 \\
& - \frac{3}{2}\omega\mathrm{Ct}_{66}\epsilon_{11}h^2 - \frac{1}{4}\omega\mathrm{Cc}_{12}\epsilon_{22}h^2 - \frac{1}{4}\omega\mathrm{Cc}_{22}\epsilon_{22}h^2 - \frac{1}{2}\omega\mathrm{Ct}_{12}\epsilon_{22}h^2 - \frac{1}{2}\omega\mathrm{Ct}_{22}\epsilon_{22}h^2 - \frac{3}{2}\omega\mathrm{Ct}_{66}\epsilon_{22}h^2 + \frac{1}{4\mathrm{Cc}_{33}}(\omega\mathrm{Cc}_{23}^2\epsilon_{22}h^2 + \omega\mathrm{Cc}_{13}\mathrm{Cc}_{23}\epsilon_{22}h^2 \\
& + \mathrm{Cc}_{23}^2\gamma_{12}\kappa_{22}h^2 + \mathrm{Cc}_{13}\mathrm{Cc}_{23}\gamma_{12}\kappa_{22}h^2 + \omega\mathrm{Cc}_{23}^2h^2 + \omega\mathrm{Cc}_{13}\mathrm{Cc}_{23}h^2) + \frac{1}{2\mathrm{Ct}_{33}}(\omega\mathrm{Ct}_{23}^2\epsilon_{22}h^2 + \omega\mathrm{Ct}_{13}\mathrm{Ct}_{23}\epsilon_{22}h^2) - \frac{3}{2}\mathrm{Ct}_{66}\gamma_{12}\kappa_{11}h^2 - \frac{1}{4}\mathrm{Cc}_{12}\gamma_{12}\kappa_{22}h^2 \\
& - \frac{1}{4}\mathrm{Cc}_{22}\gamma_{12}\kappa_{22}h^2 - \frac{1}{2}\mathrm{Ct}_{12}\gamma_{12}\kappa_{22}h^2 - \frac{1}{2}\mathrm{Ct}_{22}\gamma_{12}\kappa_{22}h^2 - \frac{3}{2}\mathrm{Ct}_{66}\gamma_{12}\kappa_{22}h^2 + \frac{1}{2\mathrm{Ct}_{33}}(\mathrm{Ct}_{23}^2\gamma_{12}\kappa_{22}h^2 + \mathrm{Ct}_{13}\mathrm{Ct}_{23}\gamma_{12}\kappa_{22}h^2 + \omega\mathrm{Ct}_{23}^2h^2 + \omega\mathrm{Ct}_{13}\mathrm{Ct}_{23}h^2) \\
& + \frac{1}{128}(32\mathrm{Cc}_{22}\gamma_{12} + 128\mathrm{Ct}_{66}\gamma_{12} + 64\mathrm{Ct}_{66}\epsilon_{11}\gamma_{12} + 32\mathrm{Cc}_{22}\epsilon_{22}\gamma_{12} + 64\mathrm{Ct}_{66}\epsilon_{22}\gamma_{12} - \frac{1}{3\mathrm{Cc}_{33}}(4\mathrm{Cc}_{13}(24\mathrm{Ct}_{23} + 24\mathrm{Cc}_{23}\epsilon_{22})\gamma_{12} - 4\mathrm{Cc}_{23}(24\mathrm{Ct}_{23} \\
& + 24\mathrm{Cc}_{23}\epsilon_{22})\gamma_{12}) + 64\mathrm{Ct}_{66}\gamma_{13}\gamma_{23} + 2\mathrm{Cc}_{12}(16\epsilon_{22}\gamma_{12} + 16\gamma_{12}) + \frac{1}{9\mathrm{Cc}_{33}}(288\gamma_{12}\epsilon_{22}\mathrm{Cc}_{23}^2 + 288\mathrm{Cc}_{13}\gamma_{12}\epsilon_{22}\mathrm{Cc}_{23} + 48\mathrm{Ct}_{23}(6\mathrm{Cc}_{13}\gamma_{12} + 6\mathrm{Cc}_{23}\gamma_{12}) \\
& - \frac{1}{3\mathrm{Cc}_{33}}(16\mathrm{Cc}_{23}(6\mathrm{Cc}_{13}\gamma_{12} + 6\mathrm{Cc}_{23}\gamma_{12}) - 16\mathrm{Cc}_{23}(6\mathrm{Cc}_{13}\gamma_{12} + 6\mathrm{Cc}_{23}\gamma_{12})\epsilon_{22})h + \frac{1}{64}(32\mathrm{Ct}_{22}\gamma_{12} + 128\mathrm{Ct}_{66}\gamma_{12} + 64\mathrm{Ct}_{66}\epsilon_{11}\gamma_{12} + 32\mathrm{Ct}_{22}\epsilon_{22}\gamma_{12} \\
& + 64\mathrm{Ct}_{66}\epsilon_{22}\gamma_{12} - \frac{1}{3\mathrm{Ct}_{33}}4\mathrm{Ct}_{13}(24\epsilon_{22}\mathrm{Ct}_{23} + 24\mathrm{Ct}_{23})\gamma_{12} - 4\mathrm{Ct}_{23}(24\epsilon_{22}\mathrm{Ct}_{23} + 24\mathrm{Ct}_{23})\gamma_{12}) + 64\mathrm{Ct}_{66}\gamma_{13}\gamma_{23} + 2\mathrm{Ct}_{12}(16\epsilon_{22}\gamma_{12} + 16\gamma_{12}) \\
& + \frac{1}{9\mathrm{Ct}_{33}}(12\mathrm{Ct}_{13}\gamma_{12}(24\epsilon_{22}\mathrm{Ct}_{23} + 24\mathrm{Ct}_{23}) + 12\mathrm{Ct}_{23}\gamma_{12}(24\epsilon_{22}\mathrm{Ct}_{23} + 24\mathrm{Ct}_{23})) - \frac{1}{3\mathrm{Ct}_{33}}(16\mathrm{Ct}_{23}(6\mathrm{Ct}_{13}\gamma_{12}) - 16\mathrm{Ct}_{23}(6\mathrm{Ct}_{13}\gamma_{12} + 6\mathrm{Ct}_{23}\gamma_{12})\epsilon_{22}))h)
\end{aligned}
$$

$$
\begin{aligned}
\overline{A_{33}} = {} & \frac{1}{384\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h(\mathrm{Cc}_{33}(-2(39h^2\omega^2 - 72h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{13}^2 + 48\epsilon_{11}^2 + 52h^2\kappa_{11}^2 + 96\epsilon_{11} - 96h\kappa_{11} - 96h\epsilon_{11}\kappa_{11})\mathrm{Ct}_{13}^2 - 4\mathrm{Ct}_{23}(39h^2\omega^2 \\
& - 72h\gamma_{12}\omega + 36\gamma_{12}^2 + 24\gamma_{13}^2 + 24\gamma_{23}^2 + 24\epsilon_{11}^2 + 24\epsilon_{22}^2 + 26h^2\kappa_{11}^2 + 26h^2\kappa_{22}^2 + 48\epsilon_{11} + 48\epsilon_{22} - 48h\kappa_{11} - 48h\epsilon_{11}\kappa_{11} - 48h\kappa_{22} - 48h\epsilon_{22}\kappa_{22})\mathrm{Ct}_{13} \\
& - 2\mathrm{Ct}_{23}^2(39h^2\omega^2 - 72h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{23}^2 + 48\epsilon_{22}^2 + 52h^2\kappa_{22}^2 + 96\epsilon_{22} - 96h\kappa_{22} - 96h\epsilon_{22}\kappa_{22}) + \mathrm{Ct}_{33}(104\mathrm{Ct}_{11}\kappa_{11}^2h^2 + 104\mathrm{Ct}_{12}\kappa_{11}^2h^2 \\
& + 312\mathrm{Ct}_{66}\kappa_{11}^2h^2 + 52\mathrm{Cc}_{22}\kappa_{22}^2h^2 + 104\mathrm{Ct}_{12}\kappa_{22}^2h^2 + 104\mathrm{Ct}_{22}\kappa_{22}^2h^2 + 312\mathrm{Ct}_{66}\kappa_{22}^2h^2 + 39\omega^2\mathrm{Cc}_{22}h^2 + 78\omega^2\mathrm{Ct}_{11}h^2 + 156\omega^2\mathrm{Ct}_{12}h^2 + 78\omega^2\mathrm{Ct}_{22}h^2 \\
& + 624\mathrm{Ct}_{66}\kappa_{11}\kappa_{22}h^2 - 72\omega\mathrm{Cc}_{22}\gamma_{12}h - 144\omega\mathrm{Ct}_{11}\gamma_{12}h - 288\omega\mathrm{Ct}_{12}\gamma_{12}h - 144\omega\mathrm{Ct}_{22}\gamma_{12}h - 192\mathrm{Ct}_{11}\kappa_{11}h - 192\mathrm{Ct}_{12}\kappa_{11}h - 1152\mathrm{Ct}_{66}\kappa_{11}h \\
& - 192\mathrm{Ct}_{11}\epsilon_{11}\kappa_{11}h - 192\mathrm{Ct}_{12}\epsilon_{11}\kappa_{11}h - 576\mathrm{Ct}_{66}\epsilon_{11}\kappa_{11}h - 576\mathrm{Ct}_{66}\epsilon_{22}\kappa_{11}h - 96\mathrm{Cc}_{22}\kappa_{22}h - 192\mathrm{Ct}_{12}\kappa_{22}h - 192\mathrm{Ct}_{22}\kappa_{22}h - 1152\mathrm{Ct}_{66}\kappa_{22}h \\
& - 576\mathrm{Ct}_{66}\epsilon_{11}\kappa_{22}h - 96\mathrm{Cc}_{22}\epsilon_{22}\kappa_{22}h - 192\mathrm{Ct}_{12}\epsilon_{22}\kappa_{22}h - 192\mathrm{Ct}_{22}\epsilon_{22}\kappa_{22}h - 576\mathrm{Ct}_{66}\epsilon_{22}\kappa_{22}h + 36\mathrm{Cc}_{22}\gamma_{12}^2 + 72\mathrm{Ct}_{11}\gamma_{12}^2 + 144\mathrm{Ct}_{12}\gamma_{12}^2 \\
& + 72\mathrm{Ct}_{22}\gamma_{12}^2 + 96\mathrm{Ct}_{11}\gamma_{13}^2 + 96\mathrm{Ct}_{12}\gamma_{13}^2 + 48\mathrm{Cc}_{22}\gamma_{23}^2 + 96\mathrm{Ct}_{12}\gamma_{23}^2 + 96\mathrm{Ct}_{22}\gamma_{23}^2 + 96\mathrm{Ct}_{11}\epsilon_{11}^2 + 96\mathrm{Ct}_{12}\epsilon_{11}^2 + 288\mathrm{Ct}_{66}\epsilon_{11}^2 + 48\mathrm{Cc}_{22}\epsilon_{22}^2 + 96\mathrm{Ct}_{12}\epsilon_{22}^2 \\
& + 96\mathrm{Ct}_{22}\epsilon_{22}^2 + 288\mathrm{Ct}_{66}\epsilon_{22}^2 + 192\mathrm{Ct}_{11}\epsilon_{11} + 192\mathrm{Ct}_{12}\epsilon_{11} + 1152\mathrm{Ct}_{66}\epsilon_{11} + 96\mathrm{Cc}_{22}\epsilon_{22} + 192\mathrm{Ct}_{12}\epsilon_{22} + 192\mathrm{Ct}_{22}\epsilon_{22} + 1152\mathrm{Ct}_{66}\epsilon_{22} + 576\mathrm{Ct}_{66}\epsilon_{11}\epsilon_{22} \\
& + \mathrm{Cc}_{11}(39h^2\omega^2 - 72h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{13}^2 + 48\epsilon_{11}^2 + 52h^2\kappa_{11}^2 + 96\epsilon_{11} - 96h\kappa_{11} - 96h\epsilon_{11}\kappa_{11}) + 2\mathrm{Cc}_{12}(39h^2\omega^2 - 72h\gamma_{12}\omega + 36\gamma_{12}^2 + 24\gamma_{13}^2 \\
& + 24\gamma_{23}^2 + 24\epsilon_{11}^2 + 24\epsilon_{22}^2 + 26h^2\kappa_{11}^2 + 26h^2\kappa_{22}^2 + 48\epsilon_{11} + 48\epsilon_{22} - 48h\kappa_{11} - 48h\epsilon_{11}\kappa_{11} - 48h\kappa_{22} - 48h\epsilon_{22}\kappa_{22}))) - (\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(\mathrm{Cc}_{13}(39h^2\omega^2 \\
& - 72h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{13}^2 + 48\epsilon_{11}^2 + 52h^2\kappa_{11}^2 + 96\epsilon_{11} - 96h\kappa_{11} - 96h\epsilon_{11}\kappa_{11}) + \mathrm{Cc}_{23}(39h^2\omega^2 - 72h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{23}^2 + 48\epsilon_{22}^2 + 52h^2\kappa_{22}^2 \\
& + 96\epsilon_{22} - 96h\kappa_{22} - 96h\epsilon_{22}\kappa_{22}))))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{B_{11}} = {} & \frac{1}{96\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^2(\mathrm{Ct}_{33}(\mathrm{Cc}_{13}(\mathrm{Cc}_{23}(12\gamma_{12}^2 + 48\gamma_{23}^2 + 60h^2\kappa_{22}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{22} - 104h\kappa_{22}\epsilon_{22} + 48\epsilon_{22}^2 + 96\epsilon_{22}) + 64\mathrm{Ct}_{13}) \\
& + \mathrm{Cc}_{13}^2(12\gamma_{12}^2 + 48\gamma_{13}^2 + 180h^2\kappa_{11}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 312h\kappa_{11} - 312h\kappa_{11}\epsilon_{11} + 144\epsilon_{11}^2 + 288\epsilon_{11} + 64) - 32\mathrm{Ct}_{13}^2) + \mathrm{Cc}_{33}(-\mathrm{Ct}_{33}(\mathrm{Cc}_{12}(12\gamma_{12}^2 \\
& + 48\gamma_{23}^2 + 60h^2\kappa_{22}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{22} - 104h\kappa_{22}\epsilon_{22} + 48\epsilon_{22}^2 + 96\epsilon_{22}) + \mathrm{Cc}_{11}(12\gamma_{12}^2 - 26\gamma_{12}h\omega + 3(16\gamma_{13}^2 + h(60h\kappa_{11}^2 + 5h\omega^2 - 104\kappa_{11}) \\
& + \epsilon_{11}(96 - 104h\kappa_{11}) + 48\epsilon_{11}^2)) + 2(3\mathrm{Ct}_{66}(12\gamma_{12}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega) + \mathrm{Ct}_{12}(12\gamma_{12}^2 + 48\gamma_{23}^2 + 60h^2\kappa_{22}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{22} - 104h\kappa_{22}\epsilon_{22} \\
& + 48\epsilon_{22}^2 + 96\epsilon_{22}) + \mathrm{Ct}_{11}(12\gamma_{12}^2 - 26\gamma_{12}h\omega + 3(16\gamma_{13}^2 + h(60h\kappa_{11}^2 + 5h\omega^2 - 104\kappa_{11}) + \epsilon_{11}(96 - 104h\kappa_{11}) + 48\epsilon_{11}^2)))) + 2\mathrm{Ct}_{13}^2(12\gamma_{12}^2 + 3(16\gamma_{13}^2 + 60h^2\kappa_{11}^2 \\
& + 5h^2\omega^2 - 104h\kappa_{11} + \epsilon_{11}(96 - 104h\kappa_{11}) + 48\epsilon_{11}^2 + 32) - 26\gamma_{12}h\omega) + 2\mathrm{Ct}_{23}\mathrm{Ct}_{13}(12\gamma_{12}^2 + 48\gamma_{23}^2 + 60h^2\kappa_{22}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{22} \\
& - 104h\kappa_{22}\epsilon_{22} + 48\epsilon_{22}^2 + 96\epsilon_{22})))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{B_{12}} = {} & \frac{1}{48\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^2(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Ct}_{66}(-36\gamma_{12}^2 - 45h^2\omega^2 + 78\gamma_{12}h\omega) - 4(\mathrm{Cc}_{12} + 2\mathrm{Ct}_{12})(h(\kappa_{11}(15h\kappa_{22} - 13) - 13\kappa_{22}) + \epsilon_{22}(12 - 13h\kappa_{11}) \\
& + \epsilon_{11}(-13h\kappa_{22} + 12\epsilon_{22} + 12))) + 8\mathrm{Ct}_{13}\mathrm{Ct}_{23}(15h^2\kappa_{11}\kappa_{22} - 13h\kappa_{11} - 13h\kappa_{22} + \epsilon_{22}(12 - 13h\kappa_{11}) + \epsilon_{11}(-13h\kappa_{22} + 12\epsilon_{22} + 12) + 12)) \\
& + 4\mathrm{Ct}_{33}(\mathrm{Cc}_{13}(\mathrm{Cc}_{23}(15h^2\kappa_{11}\kappa_{22} - 13h\kappa_{11} - 13h\kappa_{22} + \epsilon_{22}(12 - 13h\kappa_{11}) + \epsilon_{11}(-13h\kappa_{22} + 12\epsilon_{22} + 12) + 8) + 4\mathrm{Ct}_{23}) + 4\mathrm{Ct}_{13}(\mathrm{Cc}_{23} - \mathrm{Ct}_{23})))
\end{aligned}
$$

$$
\begin{aligned}
\overline{B_{13}} = {} & \frac{1}{48\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^2(\mathrm{Cc}_{13}(\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(\gamma_{12}(-13h\kappa_{11} + 12\epsilon_{11} + 12) + h\omega(15h\kappa_{11} - 13\epsilon_{11} - 13)) + \mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cc}_{11}(\gamma_{12}(13h\kappa_{11} - 12\epsilon_{11} - 12) \\
& + h\omega(-15h\kappa_{11} + 13\epsilon_{11} + 13)) + \mathrm{Cc}_{12}(\gamma_{12}(13h\kappa_{11} - 12\epsilon_{11} - 12) + h\omega(-15h\kappa_{11} + 13\epsilon_{11} + 13)) - 2(\mathrm{Ct}_{11}(\gamma_{12}(-13h\kappa_{11} + 12\epsilon_{11} + 12) \\
& + h\omega(15h\kappa_{11} - 13\epsilon_{11} - 13)) + \mathrm{Ct}_{12}(\gamma_{12}(-13h\kappa_{11} + 12\epsilon_{11} + 12) + h\omega(15h\kappa_{11} - 13\epsilon_{11} - 13)) + 3\mathrm{Ct}_{66}(12\gamma_{13}\gamma_{23} + \gamma_{12}(-13h\kappa_{11} - 13h\kappa_{22} \\
& + 12\epsilon_{11} + 12\epsilon_{22} + 24) + h\omega(15h\kappa_{11} + 15h\kappa_{22} - 13\epsilon_{11} - 13\epsilon_{22} - 26)))) + \mathrm{Ct}_{13}^2(\gamma_{12}(-26h\kappa_{11} + 24\epsilon_{11} + 24) + 2h\omega(15h\kappa_{11} - 13\epsilon_{11} - 13)) \\
& + 2\mathrm{Ct}_{23}\mathrm{Ct}_{13}(\gamma_{12}(-13h\kappa_{11} + 12\epsilon_{11} + 12) + h\omega(15h\kappa_{11} - 13\epsilon_{11} - 13))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{B_{22}} = {} & \frac{1}{96\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^2(\mathrm{Ct}_{33}(64\mathrm{Cc}_{23}\mathrm{Ct}_{23} + \mathrm{Cc}_{23}^2(12\gamma_{12}^2 + 48\gamma_{23}^2 + 180h^2\kappa_{22}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 312h\kappa_{22} - 312h\kappa_{22}\epsilon_{22} + 144\epsilon_{22}^2 + 288\epsilon_{22} + 64) \\
& + \mathrm{Cc}_{13}\mathrm{Cc}_{23}(12\gamma_{12}^2 + 48\gamma_{13}^2 + 60h^2\kappa_{11}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{11} - 104h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11}) - 32\mathrm{Ct}_{23}^2) + \mathrm{Cc}_{33}(-\mathrm{Ct}_{33}(\mathrm{Cc}_{12}(12\gamma_{12}^2 \\
& + 48\gamma_{13}^2 + 60h^2\kappa_{11}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{11} - 104h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11}) + \mathrm{Cc}_{22}(12\gamma_{12}^2 - 26\gamma_{12}h\omega + 3(16\gamma_{23}^2 + h(60h\kappa_{22}^2 + 5h\omega^2 \\
& - 104\kappa_{22}) + \epsilon_{22}(96 - 104h\kappa_{22}) + 48\epsilon_{22}^2)) + 2(3\mathrm{Ct}_{66}(12\gamma_{12}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega) + \mathrm{Ct}_{12}(12\gamma_{12}^2 + 48\gamma_{13}^2 + 60h^2\kappa_{11}^2 + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{11} \\
& - 104h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11}) + \mathrm{Ct}_{22}(12\gamma_{12}^2 - 26\gamma_{12}h\omega + 3(16\gamma_{23}^2 + h(60h\kappa_{22}^2 + 5h\omega^2 - 104\kappa_{22}) + \epsilon_{22}(96 - 104h\kappa_{22}) + 48\epsilon_{22}^2)))) + 2\mathrm{Ct}_{23}^2(12\gamma_{12}^2 \\
& + 3(16\gamma_{23}^2 + 60h^2\kappa_{22}^2 + 5h^2\omega^2 - 104h\kappa_{22} + \epsilon_{22}(96 - 104h\kappa_{22}) + 48\epsilon_{22}^2 + 32) - 26\gamma_{12}h\omega) + 2\mathrm{Ct}_{13}\mathrm{Ct}_{23}(12\gamma_{12}^2 + 48\gamma_{13}^2 + 60h^2\kappa_{11}^2 \\
& + 15h^2\omega^2 - 26\gamma_{12}h\omega - 104h\kappa_{11} - 104h\kappa_{11}\epsilon_{11} + 48\epsilon_{11}^2 + 96\epsilon_{11})))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{B_{23}} = {} & \frac{1}{48\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^2(\mathrm{Cc}_{23}(\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(\gamma_{12}(-13h\kappa_{22} + 12\epsilon_{22} + 12) + h\omega(15h\kappa_{22} - 13\epsilon_{22} - 13)) + \mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cc}_{12}(\gamma_{12}(13h\kappa_{22} - 12\epsilon_{22} - 12) \\
& + h\omega(-15h\kappa_{22} + 13\epsilon_{22} + 13)) + \mathrm{Cc}_{22}(\gamma_{12}(13h\kappa_{22} - 12\epsilon_{22} - 12) + h\omega(-15h\kappa_{22} + 13\epsilon_{22} + 13)) - 2(\mathrm{Ct}_{12}(\gamma_{12}(-13h\kappa_{22} + 12\epsilon_{22} + 12) \\
& + h\omega(15h\kappa_{22} - 13\epsilon_{22} - 13)) + \mathrm{Ct}_{22}(\gamma_{12}(-13h\kappa_{22} + 12\epsilon_{22} + 12) + h\omega(15h\kappa_{22} - 13\epsilon_{22} - 13)) + 3\mathrm{Ct}_{66}(12\gamma_{13}\gamma_{23} + \gamma_{12}(-13h\kappa_{11} - 13h\kappa_{22} + 12\epsilon_{11} \\
& + 12\epsilon_{22} + 24) + h\omega(15h\kappa_{11} + 15h\kappa_{22} - 13\epsilon_{11} - 13\epsilon_{22} - 26)))) + \mathrm{Ct}_{23}^2(\gamma_{12}(-26h\kappa_{22} + 24\epsilon_{22} + 24) + 2h\omega(15h\kappa_{22} - 13\epsilon_{22} - 13)) \\
& + 2\mathrm{Ct}_{13}\mathrm{Ct}_{23}(\gamma_{12}(-13h\kappa_{22} + 12\epsilon_{22} + 12) + h\omega(15h\kappa_{22} - 13\epsilon_{22} - 13))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{B_{33}} = {} & \frac{1}{384\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^2((\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(\mathrm{Cc}_{13}(45h^2\omega^2 - 78h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{13}^2 + 48\epsilon_{11}^2 + 60h^2\kappa_{11}^2 + 96\epsilon_{11} - 104h\kappa_{11} - 104h\epsilon_{11}\kappa_{11}) \\
& + \mathrm{Cc}_{23}(45h^2\omega^2 - 78h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{23}^2 + 48\epsilon_{22}^2 + 60h^2\kappa_{22}^2 + 96\epsilon_{22} - 104h\kappa_{22} - 104h\epsilon_{22}\kappa_{22})) + \mathrm{Cc}_{33}(2(45h^2\omega^2 - 78h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{13}^2 \\
& + 48\epsilon_{11}^2 + 60h^2\kappa_{11}^2 + 96\epsilon_{11} - 104h\kappa_{11} - 104h\epsilon_{11}\kappa_{11})\mathrm{Ct}_{13}^2 + 4\mathrm{Ct}_{23}(45h^2\omega^2 - 78h\gamma_{12}\omega + 36\gamma_{12}^2 + 24\gamma_{13}^2 + 24\gamma_{23}^2 + 24\epsilon_{11}^2 + 24\epsilon_{22}^2 + 30h^2\kappa_{11}^2 \\
& + 30h^2\kappa_{22}^2 + 48\epsilon_{11} + 48\epsilon_{22} - 52h\kappa_{11} - 52h\epsilon_{11}\kappa_{11} - 52h\kappa_{22} - 52h\epsilon_{22}\kappa_{22})\mathrm{Ct}_{13} + 2\mathrm{Ct}_{23}^2(45h^2\omega^2 - 78h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{23}^2 + 48\epsilon_{22}^2 + 60h^2\kappa_{22}^2 \\
& + 96\epsilon_{22} - 104h\kappa_{22} - 104h\epsilon_{22}\kappa_{22}) - \mathrm{Ct}_{33}(120\mathrm{Ct}_{11}\kappa_{11}^2h^2 + 120\mathrm{Ct}_{12}\kappa_{11}^2h^2 + 360\mathrm{Ct}_{66}\kappa_{11}^2h^2 + 60\mathrm{Cc}_{22}\kappa_{22}^2h^2 + 120\mathrm{Ct}_{12}\kappa_{22}^2h^2 + 120\mathrm{Ct}_{22}\kappa_{22}^2h^2 \\
& + 360\mathrm{Ct}_{66}\kappa_{22}^2h^2 + 45\omega^2\mathrm{Cc}_{22}h^2 + 90\omega^2\mathrm{Ct}_{11}h^2 + 180\omega^2\mathrm{Ct}_{12}h^2 + 90\omega^2\mathrm{Ct}_{22}h^2 + 720\mathrm{Ct}_{66}\kappa_{11}\kappa_{22}h^2 - 78\omega\mathrm{Cc}_{22}\gamma_{12}h - 156\omega\mathrm{Ct}_{11}\gamma_{12}h \\
& - 312\omega\mathrm{Ct}_{12}\gamma_{12}h - 156\omega\mathrm{Ct}_{22}\gamma_{12}h - 208\mathrm{Ct}_{11}\kappa_{11}h - 208\mathrm{Ct}_{12}\kappa_{11}h - 1248\mathrm{Ct}_{66}\kappa_{11}h - 208\mathrm{Ct}_{11}\epsilon_{11}\kappa_{11}h - 208\mathrm{Ct}_{12}\epsilon_{11}\kappa_{11}h - 624\mathrm{Ct}_{66}\epsilon_{11}\kappa_{11}h \\
& - 624\mathrm{Ct}_{66}\epsilon_{22}\kappa_{11}h - 104\mathrm{Cc}_{22}\kappa_{22}h - 208\mathrm{Ct}_{12}\kappa_{22}h - 208\mathrm{Ct}_{22}\kappa_{22}h - 1248\mathrm{Ct}_{66}\kappa_{22}h - 624\mathrm{Ct}_{66}\epsilon_{11}\kappa_{22}h - 104\mathrm{Cc}_{22}\epsilon_{22}\kappa_{22}h - 208\mathrm{Ct}_{12}\epsilon_{22}\kappa_{22}h \\
& - 208\mathrm{Ct}_{22}\epsilon_{22}\kappa_{22}h - 624\mathrm{Ct}_{66}\epsilon_{22}\kappa_{22}h + 36\mathrm{Cc}_{22}\gamma_{12}^2 + 72\mathrm{Ct}_{11}\gamma_{12}^2 + 144\mathrm{Ct}_{12}\gamma_{12}^2 + 72\mathrm{Ct}_{22}\gamma_{12}^2 + 96\mathrm{Ct}_{11}\gamma_{13}^2 + 96\mathrm{Ct}_{12}\gamma_{13}^2 + 48\mathrm{Cc}_{22}\gamma_{23}^2 + 96\mathrm{Ct}_{12}\gamma_{23}^2 \\
& + 96\mathrm{Ct}_{22}\gamma_{23}^2 + 96\mathrm{Ct}_{11}\epsilon_{11}^2 + 96\mathrm{Ct}_{12}\epsilon_{11}^2 + 288\mathrm{Ct}_{66}\epsilon_{11}^2 + 48\mathrm{Cc}_{22}\epsilon_{22}^2 + 96\mathrm{Ct}_{12}\epsilon_{22}^2 + 96\mathrm{Ct}_{22}\epsilon_{22}^2 + 288\mathrm{Ct}_{66}\epsilon_{22}^2 + 192\mathrm{Ct}_{11}\epsilon_{11} + 192\mathrm{Ct}_{12}\epsilon_{11} \\
& + 1152\mathrm{Ct}_{66}\epsilon_{11} + 96\mathrm{Cc}_{22}\epsilon_{22} + 192\mathrm{Ct}_{12}\epsilon_{22} + 192\mathrm{Ct}_{22}\epsilon_{22} + 1152\mathrm{Ct}_{66}\epsilon_{22} + 576\mathrm{Ct}_{66}\epsilon_{11}\epsilon_{22} + \mathrm{Cc}_{11}(45h^2\omega^2 - 78h\gamma_{12}\omega + 36\gamma_{12}^2 + 48\gamma_{13}^2 + 48\epsilon_{11}^2 \\
& + 60h^2\kappa_{11}^2 + 96\epsilon_{11} - 104h\kappa_{11} - 104h\epsilon_{11}\kappa_{11}) + 2\mathrm{Cc}_{12}(45h^2\omega^2 - 78h\gamma_{12}\omega + 36\gamma_{12}^2 + 24\gamma_{13}^2 + 24\gamma_{23}^2 + 24\epsilon_{11}^2 + 24\epsilon_{22}^2 + 30h^2\kappa_{11}^2 + 30h^2\kappa_{22}^2 + 48\epsilon_{11} \\
& + 48\epsilon_{22} - 52h\kappa_{11} - 52h\epsilon_{11}\kappa_{11} - 52h\kappa_{22} - 52h\epsilon_{22}\kappa_{22}))))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{D_{11}} = {} & \frac{1}{17280\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(\mathrm{Cc}_{13}^2(12800\mathrm{Cc}_{33} - \mathrm{Ct}_{33}(2340\gamma_{12}^2 + 9360\gamma_{13}^2 + 39204h^2\kappa_{11}^2 + 3267h^2\omega^2 - 5400\gamma_{12}h\omega - 64800h\kappa_{11} - 64800h\kappa_{11}\epsilon_{11} + 28080\epsilon_{11}^2 \\
& + 56160\epsilon_{11} + 16640)) - \mathrm{Cc}_{13}(\mathrm{Ct}_{33}(9\mathrm{Cc}_{23}(260\gamma_{12}^2 + 1040\gamma_{23}^2 + 1452h^2\kappa_{22}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{22} - 2400h\kappa_{22}\epsilon_{22} + 1040\epsilon_{22}^2 + 2080\epsilon_{22}) \\
& + 4160\mathrm{Ct}_{13}) + 25600\mathrm{Cc}_{33}\mathrm{Ct}_{13}) + \mathrm{Cc}_{33}(9\mathrm{Ct}_{33}(\mathrm{Cc}_{11}(260\gamma_{12}^2 + 1040\gamma_{13}^2 + 4356h^2\kappa_{11}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 7200h\kappa_{11} - 7200h\kappa_{11}\epsilon_{11} + 3120\epsilon_{11}^2 \\
& + 6240\epsilon_{11}) + \mathrm{Cc}_{12}(260\gamma_{12}^2 + 1040\gamma_{23}^2 + 1452h^2\kappa_{22}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{22} - 2400h\kappa_{22}\epsilon_{22} + 1040\epsilon_{22}^2 + 2080\epsilon_{22}) + 2(3\mathrm{Ct}_{66}(260\gamma_{12}^2 \\
& + 363h^2\omega^2 - 600\gamma_{12}h\omega) + \mathrm{Ct}_{11}(260\gamma_{12}^2 + 1040\gamma_{13}^2 + 4356h^2\kappa_{11}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 7200h\kappa_{11} - 7200h\kappa_{11}\epsilon_{11} + 3120\epsilon_{11}^2 + 6240\epsilon_{11}) \\
& + \mathrm{Ct}_{12}(260\gamma_{12}^2 + 1040\gamma_{23}^2 + 1452h^2\kappa_{22}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{22} - 2400h\kappa_{22}\epsilon_{22} + 1040\epsilon_{22}^2 + 2080\epsilon_{22}))) - 2\mathrm{Ct}_{13}^2(2340\gamma_{12}^2 + 9360\gamma_{13}^2 \\
& + 39204h^2\kappa_{11}^2 + 3267h^2\omega^2 - 5400\gamma_{12}h\omega - 64800h\kappa_{11} - 64800h\kappa_{11}\epsilon_{11} + 28080\epsilon_{11}^2 + 56160\epsilon_{11} + 12320) - 18\mathrm{Ct}_{23}\mathrm{Ct}_{13}(260\gamma_{12}^2 + 1040\gamma_{23}^2 \\
& + 1452h^2\kappa_{22}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{22} - 2400h\kappa_{22}\epsilon_{22} + 1040\epsilon_{22}^2 + 2080\epsilon_{22})) + 2080\mathrm{Ct}_{13}^2\mathrm{Ct}_{33})))
\end{aligned}
$$

$$
\begin{aligned}
\overline{D_{12}} = {} & -\frac{1}{8640\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(-7020\gamma_{12}^2\mathrm{Cc}_{33}\mathrm{Ct}_{33}\mathrm{Ct}_{66} + 26136\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}h^2\kappa_{11}\kappa_{22} - 13068\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h^2\kappa_{11}\kappa_{22} - 26136\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}h^2\kappa_{11}\kappa_{22} \\
& - 9801\mathrm{Cc}_{33}\mathrm{Ct}_{33}\mathrm{Ct}_{66}h^2\omega^2 - 4\mathrm{Cc}_{13}(\mathrm{Cc}_{23}(1600\mathrm{Cc}_{33} + \mathrm{Ct}_{33}(-3267h^2\kappa_{11}\kappa_{22} + 2700h\kappa_{11} + 2700h\kappa_{22} + 180\epsilon_{22}(15h\kappa_{11} - 13) - 180\epsilon_{11}(-15h\kappa_{22} \\
& + 13\epsilon_{22} + 13) - 2080)) - 20\mathrm{Ct}_{23}(80\mathrm{Cc}_{33} + 13\mathrm{Ct}_{33})) + 16200\gamma_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}\mathrm{Ct}_{66}h\omega - 21600\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}h\kappa_{11} + 10800\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h\kappa_{11} \\
& + 21600\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}h\kappa_{11} - 21600\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}h\kappa_{22} + 10800\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h\kappa_{22} + 21600\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}h\kappa_{22} - 21600\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}h\kappa_{11}\epsilon_{22} \\
& + 10800\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h\kappa_{11}\epsilon_{22} + 21600\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}h\kappa_{11}\epsilon_{22} - 21600\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}h\kappa_{22}\epsilon_{11} + 10800\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}h\kappa_{22}\epsilon_{11} \\
& + 21600\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}h\kappa_{22}\epsilon_{11} + 18720\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}\epsilon_{11} - 9360\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}\epsilon_{11} - 18720\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}\epsilon_{11} + 18720\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}\epsilon_{22} \\
& - 9360\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}\epsilon_{22} - 18720\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}\epsilon_{22} + 18720\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23}\epsilon_{11}\epsilon_{22} - 9360\mathrm{Cc}_{12}\mathrm{Cc}_{33}\mathrm{Ct}_{33}\epsilon_{11}\epsilon_{22} - 18720\mathrm{Cc}_{33}\mathrm{Ct}_{12}\mathrm{Ct}_{33}\epsilon_{11}\epsilon_{22} \\
& + 12320\mathrm{Cc}_{33}\mathrm{Ct}_{13}\mathrm{Ct}_{23} + 80\mathrm{Cc}_{23}\mathrm{Ct}_{13}(80\mathrm{Cc}_{33} + 13\mathrm{Ct}_{33}) - 1040\mathrm{Ct}_{13}\mathrm{Ct}_{23}\mathrm{Ct}_{33}))
\end{aligned}
$$

$$
\begin{aligned}
\overline{D_{13}} = {} & \frac{1}{960\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cc}_{11}(20\gamma_{12}(-15h\kappa_{11} + 13\epsilon_{11} + 13) + 3h\omega(121h\kappa_{11} - 100\epsilon_{11} - 100)) + \mathrm{Cc}_{12}(20\gamma_{12}(-15h\kappa_{11} + 13\epsilon_{11} + 13) \\
& + 3h\omega(121h\kappa_{11} - 100\epsilon_{11} - 100)) + 2(\mathrm{Ct}_{11}(20\gamma_{12}(-15h\kappa_{11} + 13\epsilon_{11} + 13) + 3h\omega(121h\kappa_{11} - 100\epsilon_{11} - 100)) + \mathrm{Ct}_{12}(20\gamma_{12}(-15h\kappa_{11} + 13\epsilon_{11} + 13) \\
& + 3h\omega(121h\kappa_{11} - 100\epsilon_{11} - 100)) + 3\mathrm{Ct}_{66}(260\gamma_{13}\gamma_{23} + 20\gamma_{12}(-15h\kappa_{11} - 15h\kappa_{22} + 13\epsilon_{11} + 13\epsilon_{22} + 26) + 3h\omega(121h\kappa_{11} + 121h\kappa_{22} - 100\epsilon_{11} \\
& - 100\epsilon_{22} - 200)))) + \mathrm{Ct}_{13}^2(6h\omega(-121h\kappa_{11} + 100\epsilon_{11} + 100) - 40\gamma_{12}(-15h\kappa_{11} + 13\epsilon_{11} + 13)) + 2\mathrm{Ct}_{23}\mathrm{Ct}_{13}(3h\omega(-121h\kappa_{11} + 100\epsilon_{11} + 100) \\
& - 20\gamma_{12}(-15h\kappa_{11} + 13\epsilon_{11} + 13))) - \mathrm{Cc}_{13}(\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(20\gamma_{12}(-15h\kappa_{11} + 13\epsilon_{11} + 13) + 3h\omega(121h\kappa_{11} - 100\epsilon_{11} - 100)))
\end{aligned}
$$

$$
\begin{aligned}
\overline{D_{22}} = {} & \frac{1}{17280\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(\mathrm{Cc}_{23}^2(12800\mathrm{Cc}_{33} - \mathrm{Ct}_{33}(2340\gamma_{12}^2 + 9360\gamma_{23}^2 + 39204h^2\kappa_{22}^2 + 3267h^2\omega^2 - 5400\gamma_{12}h\omega - 64800h\kappa_{22} - 64800h\kappa_{22}\epsilon_{22} \\
& + 28080\epsilon_{22}^2 + 56160\epsilon_{22} + 16640)) - \mathrm{Cc}_{23}(\mathrm{Ct}_{33}(9\mathrm{Cc}_{13}(260\gamma_{12}^2 + 1040\gamma_{13}^2 + 1452h^2\kappa_{11}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{11} - 2400h\kappa_{11}\epsilon_{11} \\
& + 1040\epsilon_{11}^2 + 2080\epsilon_{11}) + 4160\mathrm{Ct}_{23}) + 25600\mathrm{Cc}_{33}\mathrm{Ct}_{23}) + \mathrm{Cc}_{33}(9\mathrm{Ct}_{33}(\mathrm{Cc}_{12}(260\gamma_{12}^2 + 1040\gamma_{13}^2 + 1452h^2\kappa_{11}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{11} \\
& - 2400h\kappa_{11}\epsilon_{11} + 1040\epsilon_{11}^2 + 2080\epsilon_{11}) + \mathrm{Cc}_{22}(260\gamma_{12}^2 + 1040\gamma_{23}^2 + 4356h^2\kappa_{22}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 7200h\kappa_{22} - 7200h\kappa_{22}\epsilon_{22} + 3120\epsilon_{22}^2 \\
& + 6240\epsilon_{22}) + 2(3\mathrm{Ct}_{66}(260\gamma_{12}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega) + \mathrm{Ct}_{12}(260\gamma_{12}^2 + 1040\gamma_{13}^2 + 1452h^2\kappa_{11}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{11} - 2400h\kappa_{11}\epsilon_{11} \\
& + 1040\epsilon_{11}^2 + 2080\epsilon_{11}) + \mathrm{Ct}_{22}(260\gamma_{12}^2 + 1040\gamma_{23}^2 + 4356h^2\kappa_{22}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 7200h\kappa_{22} - 7200h\kappa_{22}\epsilon_{22} + 3120\epsilon_{22}^2 + 6240\epsilon_{22}))) \\
& - 2\mathrm{Ct}_{23}^2(2340\gamma_{12}^2 + 9360\gamma_{23}^2 + 39204h^2\kappa_{22}^2 + 3267h^2\omega^2 - 5400\gamma_{12}h\omega - 64800h\kappa_{22} - 64800h\kappa_{22}\epsilon_{22} + 28080\epsilon_{22}^2 + 56160\epsilon_{22} + 12320) \\
& - 18\mathrm{Ct}_{13}\mathrm{Ct}_{23}(260\gamma_{12}^2 + 1040\gamma_{13}^2 + 1452h^2\kappa_{11}^2 + 363h^2\omega^2 - 600\gamma_{12}h\omega - 2400h\kappa_{11} - 2400h\kappa_{11}\epsilon_{11} + 1040\epsilon_{11}^2 + 2080\epsilon_{11})) + 2080\mathrm{Ct}_{23}^2\mathrm{Ct}_{33})))
\end{aligned}
$$

$$
\begin{aligned}
\overline{D_{23}} = {} & \frac{1}{960\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(\mathrm{Cc}_{12}(20\gamma_{12}(-15h\kappa_{22} + 13\epsilon_{22} + 13) + 3h\omega(121h\kappa_{22} - 100\epsilon_{22} - 100)) + \mathrm{Cc}_{22}(20\gamma_{12}(-15h\kappa_{22} + 13\epsilon_{22} + 13) \\
& + 3h\omega(121h\kappa_{22} - 100\epsilon_{22} - 100)) + 2(\mathrm{Ct}_{12}(20\gamma_{12}(-15h\kappa_{22} + 13\epsilon_{22} + 13) + 3h\omega(121h\kappa_{22} - 100\epsilon_{22} - 100)) + \mathrm{Ct}_{22}(20\gamma_{12}(-15h\kappa_{22} + 13\epsilon_{22} + 13) \\
& + 3h\omega(121h\kappa_{22} - 100\epsilon_{22} - 100)) + 3\mathrm{Ct}_{66}(260\gamma_{13}\gamma_{23} + 20\gamma_{12}(-15h\kappa_{11} - 15h\kappa_{22} + 13\epsilon_{11} + 13\epsilon_{22} + 26) + 3h\omega(121h\kappa_{11} + 121h\kappa_{22} - 100\epsilon_{11} \\
& - 100\epsilon_{22} - 200)))) + \mathrm{Ct}_{23}^2(6h\omega(-121h\kappa_{22} + 100\epsilon_{22} + 100) - 40\gamma_{12}(-15h\kappa_{22} + 13\epsilon_{22} + 13)) + 2\mathrm{Ct}_{13}\mathrm{Ct}_{23}(3h\omega(-121h\kappa_{22} + 100\epsilon_{22} + 100) \\
& - 20\gamma_{12}(-15h\kappa_{22} + 13\epsilon_{22} + 13))) - \mathrm{Cc}_{23}(\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(20\gamma_{12}(-15h\kappa_{22} + 13\epsilon_{22} + 13) + 3h\omega(121h\kappa_{22} - 100\epsilon_{22} - 100)))
\end{aligned}
$$

$$
\begin{aligned}
\overline{D_{33}} = {} & \frac{1}{7680\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h^3(\mathrm{Cc}_{33}(-2(1089h^2\omega^2 - 1800h\gamma_{12}\omega + 780\gamma_{12}^2 + 1040\gamma_{13}^2 + 1040\epsilon_{11}^2 + 1452h^2\kappa_{11}^2 + 2080\epsilon_{11} - 2400h\kappa_{11} - 2400h\epsilon_{11}\kappa_{11})\mathrm{Ct}_{13}^2 \\
& - 4\mathrm{Ct}_{23}(1089h^2\omega^2 - 1800h\gamma_{12}\omega + 780\gamma_{12}^2 + 520\gamma_{13}^2 + 520\gamma_{23}^2 + 520\epsilon_{11}^2 + 520\epsilon_{22}^2 + 726h^2\kappa_{11}^2 + 726h^2\kappa_{22}^2 + 1040\epsilon_{11} + 1040\epsilon_{22} - 1200h\kappa_{11} \\
& - 1200h\epsilon_{11}\kappa_{11} - 1200h\kappa_{22} - 1200h\epsilon_{22}\kappa_{22})\mathrm{Ct}_{13} - 2\mathrm{Ct}_{23}^2(1089h^2\omega^2 - 1800h\gamma_{12}\omega + 780\gamma_{12}^2 + 1040\gamma_{23}^2 + 1040\epsilon_{22}^2 + 1452h^2\kappa_{22}^2 + 2080\epsilon_{22} \\
& - 2400h\kappa_{22} - 2400h\epsilon_{22}\kappa_{22}) + \mathrm{Ct}_{33}(2904\mathrm{Ct}_{11}\kappa_{11}^2h^2 + 2904\mathrm{Ct}_{12}\kappa_{11}^2h^2 + 8712\mathrm{Ct}_{66}\kappa_{11}^2h^2 + 1452\mathrm{Cc}_{22}\kappa_{22}^2h^2 + 2904\mathrm{Ct}_{12}\kappa_{22}^2h^2 + 2904\mathrm{Ct}_{22}\kappa_{22}^2h^2 \\
& + 8712\mathrm{Ct}_{66}\kappa_{22}^2h^2 + 1089\omega^2\mathrm{Cc}_{22}h^2 + 2178\omega^2\mathrm{Ct}_{11}h^2 + 4356\omega^2\mathrm{Ct}_{12}h^2 + 2178\omega^2\mathrm{Ct}_{22}h^2 + 17424\mathrm{Ct}_{66}\kappa_{11}\kappa_{22}h^2 - 1800\omega\mathrm{Cc}_{22}\gamma_{12}h \\
& - 3600\omega\mathrm{Ct}_{11}\gamma_{12}h - 7200\omega\mathrm{Ct}_{12}\gamma_{12}h - 3600\omega\mathrm{Ct}_{22}\gamma_{12}h - 4800\mathrm{Ct}_{11}\kappa_{11}h - 4800\mathrm{Ct}_{12}\kappa_{11}h - 28800\mathrm{Ct}_{66}\kappa_{11}h - 4800\mathrm{Ct}_{11}\epsilon_{11}\kappa_{11}h \\
& - 4800\mathrm{Ct}_{12}\epsilon_{11}\kappa_{11}h - 14400\mathrm{Ct}_{66}\epsilon_{11}\kappa_{11}h - 14400\mathrm{Ct}_{66}\epsilon_{22}\kappa_{11}h - 2400\mathrm{Cc}_{22}\kappa_{22}h - 4800\mathrm{Ct}_{12}\kappa_{22}h - 4800\mathrm{Ct}_{22}\kappa_{22}h - 28800\mathrm{Ct}_{66}\kappa_{22}h \\
& - 14400\mathrm{Ct}_{66}\epsilon_{11}\kappa_{22}h - 2400\mathrm{Cc}_{22}\epsilon_{22}\kappa_{22}h - 4800\mathrm{Ct}_{12}\epsilon_{22}\kappa_{22}h - 4800\mathrm{Ct}_{22}\epsilon_{22}\kappa_{22}h - 14400\mathrm{Ct}_{66}\epsilon_{22}\kappa_{22}h + 780\mathrm{Cc}_{22}\gamma_{12}^2 + 1560\mathrm{Ct}_{11}\gamma_{12}^2 \\
& + 3120\mathrm{Ct}_{12}\gamma_{12}^2 + 1560\mathrm{Ct}_{22}\gamma_{12}^2 + 2080\mathrm{Ct}_{11}\gamma_{13}^2 + 2080\mathrm{Ct}_{12}\gamma_{13}^2 + 1040\mathrm{Cc}_{22}\gamma_{23}^2 + 2080\mathrm{Ct}_{12}\gamma_{23}^2 + 2080\mathrm{Ct}_{22}\gamma_{23}^2 + 2080\mathrm{Ct}_{11}\epsilon_{11}^2 + 2080\mathrm{Ct}_{12}\epsilon_{11}^2 \\
& + 6240\mathrm{Ct}_{66}\epsilon_{11}^2 + 1040\mathrm{Cc}_{22}\epsilon_{22}^2 + 2080\mathrm{Ct}_{12}\epsilon_{22}^2 + 2080\mathrm{Ct}_{22}\epsilon_{22}^2 + 6240\mathrm{Ct}_{66}\epsilon_{22}^2 + 4160\mathrm{Ct}_{11}\epsilon_{11} + 4160\mathrm{Ct}_{12}\epsilon_{11} + 24960\mathrm{Ct}_{66}\epsilon_{11} + 2080\mathrm{Cc}_{22}\epsilon_{22} \\
& + 4160\mathrm{Ct}_{12}\epsilon_{22} + 4160\mathrm{Ct}_{22}\epsilon_{22} + 24960\mathrm{Ct}_{66}\epsilon_{22} + 12480\mathrm{Ct}_{66}\epsilon_{11}\epsilon_{22} + \mathrm{Cc}_{11}(1089h^2\omega^2 - 1800h\gamma_{12}\omega + 780\gamma_{12}^2 + 1040\gamma_{13}^2 + 1040\epsilon_{11}^2 + 1452h^2\kappa_{11}^2 \\
& + 2080\epsilon_{11} - 2400h\kappa_{11} - 2400h\epsilon_{11}\kappa_{11}) + 2\mathrm{Cc}_{12}(1089h^2\omega^2 - 1800h\gamma_{12}\omega + 780\gamma_{12}^2 + 520\gamma_{13}^2 + 520\gamma_{23}^2 + 520\epsilon_{11}^2 + 520\epsilon_{22}^2 + 726h^2\kappa_{11}^2 + 726h^2\kappa_{22}^2 \\
& + 1040\epsilon_{11} + 1040\epsilon_{22} - 1200h\kappa_{11} - 1200h\epsilon_{11}\kappa_{11} - 1200h\kappa_{22} - 1200h\epsilon_{22}\kappa_{22}))) - (\mathrm{Cc}_{13} + \mathrm{Cc}_{23})\mathrm{Ct}_{33}(\mathrm{Cc}_{13}(1089h^2\omega^2 - 1800h\gamma_{12}\omega + 780\gamma_{12}^2 \\
& + 1040\gamma_{13}^2 + 1040\epsilon_{11}^2 + 1452h^2\kappa_{11}^2 + 2080\epsilon_{11} - 2400h\kappa_{11} - 2400h\epsilon_{11}\kappa_{11}) + \mathrm{Cc}_{23}(1089h^2\omega^2 - 1800h\gamma_{12}\omega + 780\gamma_{12}^2 + 1040\gamma_{23}^2 + 1040\epsilon_{22}^2 \\
& + 1452h^2\kappa_{22}^2 + 2080\epsilon_{22} - 2400h\kappa_{22} - 2400h\epsilon_{22}\kappa_{22})))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{S_{11}} = {} & \frac{1}{288} h(\frac{181712\mathrm{Ct}_{55}^4}{\mathrm{Cc}_{55}(34\mathrm{Cc}_{55} + 47\mathrm{Ct}_{55})^2} - \frac{1}{34\mathrm{Cc}_{55} + 47\mathrm{Ct}_{55}}(37536\mathrm{Ct}_{55}^2 - 11712\mathrm{Cc}_{55}\mathrm{Ct}_{55}) + \frac{1}{(34\mathrm{Cc}_{55} + 47\mathrm{Ct}_{55})^2}(546912\mathrm{Ct}_{55}^3 + 433344\mathrm{Cc}_{55}\mathrm{Ct}_{55}^2 \\
& + 132736\mathrm{Cc}_{55}^2\mathrm{Ct}_{55}) + 72\mathrm{Ct}_{11}\gamma_{12}^2 + 72\mathrm{Ct}_{12}\gamma_{12}^2 + 864\mathrm{Ct}_{11}\gamma_{13}^2 + 288\mathrm{Ct}_{12}\gamma_{23}^2 + 864\mathrm{Ct}_{66}\gamma_{23}^2 + 288\mathrm{Ct}_{11}\epsilon_{11}^2 + 288\mathrm{Ct}_{12}\epsilon_{22}^2 + 312h^2\mathrm{Ct}_{11}\kappa_{11}^2 \\
& + 312h^2\mathrm{Ct}_{12}\kappa_{22}^2 + 78h^2\omega^2\mathrm{Ct}_{11} + 78h^2\omega^2\mathrm{Ct}_{12} - 144h\omega\mathrm{Ct}_{11}\gamma_{12} - 144h\omega\mathrm{Ct}_{12}\gamma_{12} + \frac{1}{\mathrm{Cc}_{33}}(72h\omega\mathrm{Cc}_{13}^2\gamma_{12} + 72h\omega\mathrm{Cc}_{13}\mathrm{Cc}_{23}\gamma_{12} \\
& + 288h\mathrm{Cc}_{13}^2\epsilon_{11}\kappa_{11} + 288h\mathrm{Cc}_{13}^2\kappa_{11}) + 576\mathrm{Ct}_{11}\epsilon_{11} + 576\mathrm{Ct}_{12}\epsilon_{22} - 576h\mathrm{Ct}_{11}\kappa_{11} - 576h\mathrm{Ct}_{11}\epsilon_{11}\kappa_{11} + \frac{1}{\mathrm{Ct}_{33}}(144h\omega\mathrm{Ct}_{13}^2\gamma_{12} \\
& + 144h\omega\mathrm{Ct}_{13}\mathrm{Ct}_{23}\gamma_{12} + 576h\mathrm{Ct}_{13}^2\kappa_{11}) + 3\mathrm{Cc}_{11}(13h^2\omega^2 - 24h\gamma_{12}\omega + 12\gamma_{12}^2 + 144\gamma_{13}^2 + 48\epsilon_{11}^2 + 52h^2\kappa_{11}^2 + 96\epsilon_{11} - 96h\kappa_{11} - 96h\epsilon_{11}\kappa_{11}) \\
& - 576h\mathrm{Ct}_{12}\kappa_{22} - 576h\mathrm{Ct}_{12}\epsilon_{22}\kappa_{22} - \frac{1}{\mathrm{Cc}_{33}}(39h^2\omega^2\mathrm{Cc}_{13}^2 - 36\mathrm{Cc}_{13}^2\gamma_{12}^2 - 36\mathrm{Cc}_{13}\mathrm{Cc}_{23}\gamma_{12}^2 - 432\mathrm{Cc}_{13}^2\gamma_{13}^2 - 144\mathrm{Cc}_{13}\mathrm{Cc}_{23}\gamma_{23}^2 - 144\mathrm{Cc}_{13}^2\epsilon_{11}^2 \\
& - 144\mathrm{Cc}_{13}\mathrm{Cc}_{23}\epsilon_{22}^2 - 156h^2\mathrm{Cc}_{13}^2\kappa_{11}^2 - 156h^2\mathrm{Cc}_{13}\mathrm{Cc}_{23}\kappa_{22}^2 - 39h^2\omega^2\mathrm{Cc}_{13}\mathrm{Cc}_{23} - 288\mathrm{Cc}_{13}^2\epsilon_{11} - 288\mathrm{Cc}_{13}\mathrm{Cc}_{23}\epsilon_{22}) - \frac{1}{\mathrm{Ct}_{33}}(78h^2\omega^2\mathrm{Ct}_{13}^2 \\
& - 72\mathrm{Ct}_{13}^2\gamma_{12}^2 - 72\mathrm{Ct}_{13}\mathrm{Ct}_{23}\gamma_{12}^2 - 864\mathrm{Ct}_{13}^2\gamma_{13}^2 - 288\mathrm{Ct}_{13}\mathrm{Ct}_{23}\gamma_{23}^2 - 288\mathrm{Ct}_{13}^2\epsilon_{11}^2 - 288\mathrm{Ct}_{13}\mathrm{Ct}_{23}\epsilon_{22}^2 - 312h^2\mathrm{Ct}_{13}^2\kappa_{11}^2 - 312h^2\mathrm{Ct}_{13}\mathrm{Ct}_{23}\kappa_{22}^2)))
\end{aligned}
$$

$$
\begin{aligned}
\overline{S_{12}} = {} & \frac{1}{8\mathrm{Cc}_{33}\mathrm{Ct}_{33}} h(\mathrm{Cc}_{33}(\mathrm{Ct}_{33}(8\gamma_{13}\gamma_{23}(\mathrm{Cc}_{12} + 2\mathrm{Ct}_{12}) + \mathrm{Ct}_{66}(48\gamma_{13}\gamma_{23} + 12\gamma_{12}(-h\kappa_{11} - h\kappa_{22} + \epsilon_{11} + \epsilon_{22} + 2) + h\omega(13h\kappa_{11} + 13h\kappa_{22} - 12\epsilon_{11} \\
& - 12\epsilon_{22} - 24))) - 16\gamma_{13}\gamma_{23}\mathrm{Ct}_{13}\mathrm{Ct}_{23}) - 8\gamma_{13}\gamma_{23}\mathrm{Cc}_{13}\mathrm{Cc}_{23}\mathrm{Ct}_{33})))))
\end{aligned}
$$

$$
\begin{aligned}
\overline{S_{22}} = {} & \frac{1}{288} h(\frac{181712\mathrm{Ct}_{44}^4}{\mathrm{Cc}_{44}(34\mathrm{Cc}_{44} + 47\mathrm{Ct}_{44})^2} + \frac{1}{(34\mathrm{Cc}_{44} + 47\mathrm{Ct}_{44})^2}(546912\mathrm{Ct}_{44}^3 + 433344\mathrm{Cc}_{44}\mathrm{Ct}_{44}^2 + 132736\mathrm{Cc}_{44}^2\mathrm{Ct}_{44}) - \frac{1}{34\mathrm{Cc}_{44} + 47\mathrm{Ct}_{44}}(37536\mathrm{Ct}_{44}^2 \\
& - 11712\mathrm{Cc}_{44}\mathrm{Ct}_{44}) + 72\mathrm{Ct}_{12}\gamma_{12}^2 + 72\mathrm{Ct}_{22}\gamma_{12}^2 + 288\mathrm{Ct}_{12}\gamma_{13}^2 + 864\mathrm{Ct}_{66}\gamma_{13}^2 + 864\mathrm{Ct}_{22}\gamma_{23}^2 + 288\mathrm{Ct}_{12}\epsilon_{11}^2 + 288\mathrm{Ct}_{22}\epsilon_{22}^2 + 312h^2\mathrm{Ct}_{12}\kappa_{11}^2 \\
& + 312h^2\mathrm{Ct}_{22}\kappa_{22}^2 + 78h^2\omega^2\mathrm{Ct}_{12} + 78h^2\omega^2\mathrm{Ct}_{22} - 144h\omega\mathrm{Ct}_{12}\gamma_{12} - 144h\omega\mathrm{Ct}_{22}\gamma_{12} + \frac{1}{\mathrm{Cc}_{33}}(72h\omega\mathrm{Cc}_{23}^2\gamma_{12} + 72h\omega\mathrm{Cc}_{13}\mathrm{Cc}_{23}\gamma_{12} \\
& + 288h\mathrm{Cc}_{13}\mathrm{Cc}_{23}\epsilon_{11}\kappa_{11} + 288h\mathrm{Cc}_{13}\mathrm{Cc}_{23}\kappa_{11}) + \frac{1}{\mathrm{Ct}_{33}}(144h\omega\mathrm{Ct}_{23}^2\gamma_{12} + 144h\omega\mathrm{Ct}_{13}\mathrm{Ct}_{23}\gamma_{12} + 576h\mathrm{Ct}_{13}\mathrm{Ct}_{23}\kappa_{11} + 576h\mathrm{Ct}_{13}\mathrm{Ct}_{23}\epsilon_{11}\kappa_{11}) \\
& + 576\mathrm{Ct}_{12}\epsilon_{11} + 576\mathrm{Ct}_{22}\epsilon_{22} - 576h\mathrm{Ct}_{12}\kappa_{11} - 576h\mathrm{Ct}_{12}\epsilon_{11}\kappa_{11} + 3\mathrm{Cc}_{12}(13h^2\omega^2 - 24h\gamma_{12}\omega + 12\gamma_{12}^2 + 48\gamma_{13}^2 + 48\epsilon_{11}^2 + 52h^2\kappa_{11}^2 + 96\epsilon_{11} \\
& - 96h\kappa_{11} - 96h\epsilon_{11}\kappa_{11}) - 576h\mathrm{Ct}_{22}\kappa_{22} - 576h\mathrm{Ct}_{22}\epsilon_{22}\kappa_{22} + 3\mathrm{Cc}_{22}(13h^2\omega^2 - 24h\gamma_{12}\omega + 12\gamma_{12}^2 + 144\gamma_{23}^2 + 48\epsilon_{22}^2 + 52h^2\kappa_{22}^2 + 96\epsilon_{22} \\
& - 96h\kappa_{22} - 96h\epsilon_{22}\kappa_{22}) - \frac{1}{\mathrm{Cc}_{33}}(39h^2\omega^2\mathrm{Cc}_{23}^2 - 36\mathrm{Cc}_{23}^2\gamma_{12}^2 - 36\mathrm{Cc}_{13}\mathrm{Cc}_{23}\gamma_{12}^2 - 144\mathrm{Cc}_{13}\mathrm{Cc}_{23}\gamma_{13}^2 - 432\mathrm{Cc}_{23}^2\gamma_{23}^2 - 144\mathrm{Cc}_{13}\mathrm{Cc}_{23}\epsilon_{11}^2 \\
& - 144\mathrm{Cc}_{23}^2\epsilon_{22}^2 - 156h^2\mathrm{Cc}_{13}\mathrm{Cc}_{23}\kappa_{11}^2 - 156h^2\mathrm{Cc}_{23}^2\kappa_{22}^2 - 39h^2\omega^2\mathrm{Cc}_{13}\mathrm{Cc}_{23} - 288\mathrm{Cc}_{13}\mathrm{Cc}_{23}\epsilon_{11} - 288\mathrm{Cc}_{23}^2\epsilon_{22}) - \frac{1}{\mathrm{Ct}_{33}}(72\mathrm{Ct}_{23}^2\gamma_{12}^2 \\
& - 72\mathrm{Ct}_{13}\mathrm{Ct}_{23}\gamma_{12}^2 - 288\mathrm{Ct}_{13}\mathrm{Ct}_{23}\gamma_{13}^2 - 864\mathrm{Ct}_{23}^2\gamma_{23}^2 - 288\mathrm{Ct}_{13}\mathrm{Ct}_{23}\epsilon_{11}^2 - 288\mathrm{Ct}_{23}^2\epsilon_{22}^2 - 312h^2\mathrm{Ct}_{13}\mathrm{Ct}_{23}\kappa_{11}^2 - 312h^2\mathrm{Ct}_{23}^2\kappa_{22}^2)))
\end{aligned}
$$

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